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DAVIES AND PECK'S UNITED COURSE.

ELEMENTARY
ARITHMETIC.

ORAL AND WRITTEN.

BY

WILLIAM G. PECK, Ph.D., L.L.D.,

Professor of Mathematics and Astronomy in Columbia College, and of
Mechanics in the School of Mines.

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SHORT COURSE IN MATHEMATICS

IN FOUR BOOKS.

ELEMENTARY ARITHMETIC.

COMPLETE ARITHMETIC.

MANUAL OF ALGEBRA.

MANUAL OF GEOMETRY.

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PREFACE.

THIS work is designed as the Introductory Volume of the Two Book Course of DAVIES and PECK. It is especially adapted to beginners. It is believed that the subjects are treated in such a manner as to interest and awaken the attention of the young.

In preparing the work, three objects have been constantly kept in view.

1. To make it educational.

2. To make it practical.

3. To adapt it to the capacity of any child whose mind is sufficiently mature to commence the study of arithmetic.

To attain these objects, every new subject has been introduced by an inductive process, and the idea thus developed has been expressed in the form of a definition. The methods and rules have been deduced from practical operations and enforced by familiar illustrations. To direct the attention to important principles, leading test questions have been freely introduced.

In determining the subjects to be included, and the space to be assigned to each, the author has been guided by a consideration of the natural development of the mental faculties. The book may be said to consist of five parts. The first part contains simple, familiar Lessons in Numbers. The second part contains the Fundamental Operations followed by General Principles and Properties of Numbers. The third contains Fractions, in which great pains have been taken to render the work intelligible to young students. Currency and the Metric System follow, because of their intimate relation to Decimal Fractions. The fourth contains Compound Numbers and Reduction. The fifth, Percentage and its applications.

The logical development of principles, the systematic arrangement of the subjects, the copiousness and variety of exercises will, it is believed, greatly aid the teacher in exciting the interest of the pupil.

Teachers who desire to give a more extended drill in the simplest operations, are referred to "PECK'S FIRST LESSONS IN NUMBERS."

To facilitate references, a complete Index to the Subjects and Definitions is inserted at the end of the volume.

The author takes great pleasure in acknowledging his obligations to many teachers who have favored him with suggestions and criticisms. But more than a passing acknowledgment is due to Prof. JOHN DUNLAP, whose long experience and superior ability as a Teacher have enabled him to render much valuable assistance in the preparation of this work.

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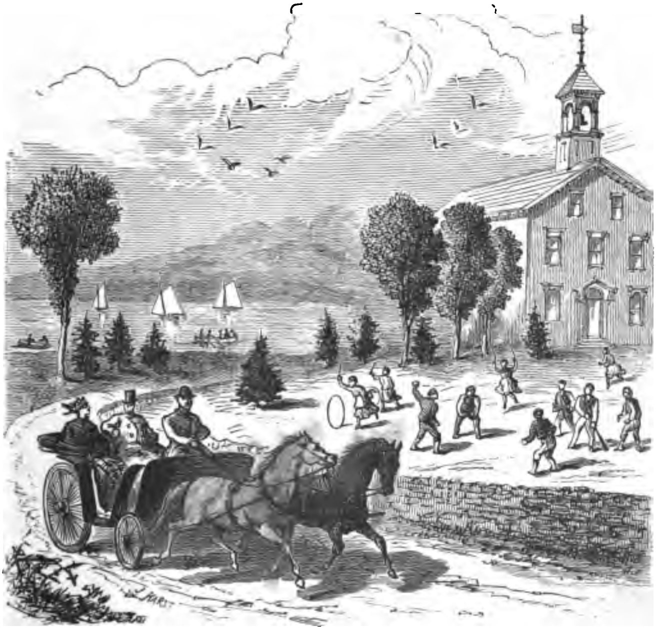
FORMATION OF NUMBERS.

LESSON I.

COUNTING.

Look at the picture and count the objects named below.



How many houses?
How many horses?
How many sail-boats?
How many high trees?
How many boats?
How many boys at play?
How many windows in front?
How many small trees?
How many birds?
How many children at play?

LESSON II.

WRITING NUMBERS. 1 TO 10.*

Write the word that tells how many houses there are in the picture. *One.* One is a **Unit.**

Write the *word* that tells how many horses. *Two.* How many ones, or units, in two?

Write the *word* that tells how many persons there are in the carriage. *Three.* How many units in three?

How many units, or ones, in four? In five? In six? In seven? In eight? In nine? How many in ten?

One, two, three, four, five, etc., are called *numbers.*

A **Number** is one or more things of the same kind.

What number tells how many girls there are on the grounds? What is the number of boys?

Thus far we have used *words* to express numbers; we may also use **Figures.**

The number of houses may be written *one* or *1;* the number of horses, *two* or *2;* the number of sail-boats, *three* or *3;* the number of girls, *four* or *4;* number of boats on the lake, *five* or *5;* number of boys, *six* or *6;* number of windows, *seven* or *7;* number of small trees, *eight* or *8;* number of birds, *nine* or *9.*

We use one more figure, *0.* It is called *naught,* and standing alone expresses no number, but is used with other figures to express numbers.

These are all the figures in use. How many are there? Write the ten figures; thus,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Read the following figures: 3, 2, 1; 4, 5, 6; 9, 8, 7, 0. Which is the least number? The greatest?

---

* See Picture, page 7.

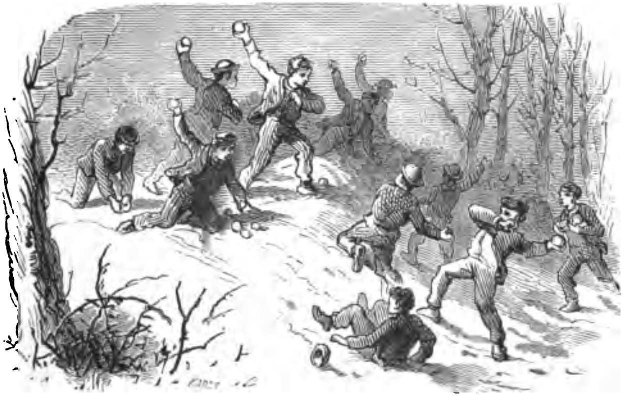
LESSON III.

NUMBERS FROM 10 TO 20.

How many figures do we use to express numbers?

The number *ten* is written by means of figures, thus, *10.*

This is one ten. How many units in one ten? Write ten.



How many boys are snow-balling?

We write the number by means of figures, thus, *11.*

The right-hand figure is 1 unit. The second figure from the right is 1 ten. Eleven is one ten and one unit. How many units in ten? How many units in eleven?

We will write, by means of figures, all the numbers from 10 to 20:

10, *11,* *12,* *13,* *14,* *15,*
ten, ***eleven,*** ***twelve,*** ***thirteen,*** ***fourteen,*** ***fifteen,***

16, *17,* *18,* *19,* *20.*
sixteen, ***seventeen,*** ***eighteen,*** ***nineteen,*** ***twenty.***

In these numbers, how many figures are used?

LESSON IV.

NUMBERS FROM 20 TO 30.

Two tens and 1 unit are twenty-one,	21.
Two tens and 2 units are twenty-two,	22.
Two tens and 3 units are twenty-three,	23.
Two tens and 4 units are twenty-four,	24.
Two tens and 5 units are twenty-five,	25.
Two tens and 6 units are twenty-six,	26.
Two tens and 7 units are twenty-seven,	27.
Two tens and 8 units are twenty-eight,	28.
Two tens and 9 units are twenty-nine,	29.

How many are one ten and 1? One ten and 2? One ten and 3? One ten and 4? One ten and 5? One ten and 6? One ten and 7? One ten and 8? One ten and 9? Two tens are how many? Two tens and 1? Two tens and 2? 2 tens and 3? 2 tens and 4? 2 tens and 5? 2 tens and 6? 2 tens and 7? 2 tens and 8? 2 tens and 9?

Write, in figures, eight, six, seventeen, nineteen, twenty-one, twenty-five, twenty-six, twenty-nine.

Write as one number, 1 ten and 7, 2 tens and 3, 1 ten and 8, 2 tens and 5, 2 tens and 7, 1 ten and 4.

Read the following numbers, 22, 19, 18, 29, 27, 16, 23, 21, 11, 12, 15, 24, 28.

One ten and one unit are how many? Two tens and two units are how many? Two tens and six units?

How many are one ten and six units? Two tens and six units? Write two tens and five units as one number.

Write three tens and six units; two tens and seven units; two tens and eight units; two tens and nine units.

LESSON V.

NUMBERS FROM 30 TO 100.

Three tens are thirty, 30. Four tens are forty, 40. Five tens are fifty, 50. Six tens are sixty, 60. Seven tens are seventy, 70. Eight tens are eighty, 80. Nine tens are ninety, 90. Ten tens are one-hundred, 100.

In writing tens we use two figures, and the second figure from the right tells how many tens we have written.

In writing 100 we use three figures, and the third figure from the right shows how many hundreds we have written.

If we use 2 instead of 1 in the third place, we have 200, (two hundred). If we use 3, we have 300; if 4, 400, and so on.

Write the numbers between 30 and 40:

Thus, *31, 32, 33, 34, 35, 36, 37, 38, 39.*

Write the numbers between 40 and 50; between 50 and 60; between 60 and 70; between 70 and 80; between 80 and 90; between 90 and 100.

Read the following numbers, 11, 14, 29, 23, 28, 31, 40, 37, 36, 42, 45, 49, 51, 53, 57, 62, 65, 69, 70, 75, 78, 82, 90, 87, 93, 71, 98, 86, 99, 100.

Four tens and 1 are how many? 4 tens and 3?

Five tens and 6 are how many? 5 tens and 7?

Six tens and 9? 6 tens and 5? 6 tens and 8?

Seven tens and 4 are how many? 7 tens and 8? 7 tens and 9?

Eight tens and 5 are how many? 8 tens and 6? 8 tens and 7?

Nine tens are how many? 9 tens and 1? 9 tens and 2? 9 tens and 3? 9 tens and 9? 9 tens and 7?

LESSON VI.

INCREASING AND DIMINISHING BY 1.

1. How many eggs are two eggs and one egg? 2 and 1, are how many? 1 and 2, are how many?

2. If we take one egg away from three eggs, how many eggs will be left? 2 from 3 leaves how many?

3. Three sheep and one sheep are how many sheep? 3 and 1, are how many? 1 and 3, are how many?

4. If we take 1 sheep away from 4 sheep, how many sheep will be left? 3 from 4 leaves how many?

5. Four birds and one bird are how many birds? 4 and 1 are how many? 1 and 4 are how many? One sheep and four sheep are how many?

6. If we take one bird from five birds, how many birds will be left? 4 from 5 leaves how many?

7. How many boys are five boys and one boy? How many are 5 and 1? 1 and 5 are how many?

8. If we take one apple from six apples, how many apples will be left? 1 from 6 leaves how many? 5 from 6 leaves how many?

9. Six chairs and one chair are how many chairs?

10. If we take one book from seven books, how many books are left? 1 from 7 leaves how many?

LESSON VII.

INCREASING AND DIMINISHING BY 2.

1. Two apples and two apples are how many apples ?

2. If we take 2 apples from 4 apples, how many apples will be left?

3. Three sheep and two sheep are how many sheep? 3 and 2 are how many ? 2 and 3 are how many?

4. If we take 2 sheep from 5 sheep, how many sheep will be left? 2 from 5 leaves how many? 3 from 5 leaves how many ?

5. How many cherries are 4 cherries and 2 cherries? 4 and 2 are how many? 2 and 4 are how many?

6. If we take 2 cherries from 6 cherries, how many cherries will be left? 2 from 6 leaves how many? 4 from 6 leaves how many?

7. How many birds are five birds and two birds? 5 and 2 are how many? 2 and 5 are how many ?

8. If we take 2 birds from 7 birds, how many birds will be left? 2 from 7 leaves how many? 5 from 7 leaves how many?

LESSON VIII.

INCREASING AND DIMINISHING BY 3.

1. Three balls and three balls are how many balls?

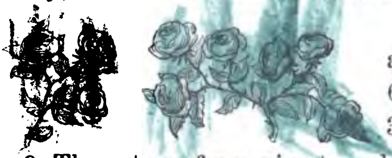
2. If we take 3 marbles from 6 marbles, how many marbles will be left? 3 from 6 leaves how many?

3. How many pears are 4 pears and 3 pears? 4 and 3 are how many? 3 and 4 are how many?

4. 3 apples from 7 apples leaves how many apples? 3 from 7 leaves how many? 4 from 7 leaves how many?

5. Five cherries and three cherries are how many cherries? 3 units and 5 units are how many units?

6. If we take three plums from 8 plums, how many plums will be left? 3 units from 8 units leaves how many units? 5 from 8 leaves how many?



7. How many roses are 6 roses and 3 roses? 6 and 3 are how many? 3 and 6 are how many?

8. Three trees from nine trees leaves how many trees? 3 from 9 leaves how many? 6 from 9 leaves how many?

9. How many apples are seven apples and three apples? H[illegible] many are 7 and 3? How many are 3 and 7?

10. If we take away 3 apples from 10 apples, how many apples will be left? 3 from 10 leaves how many? 7 from 10 leaves how many?

LESSON IX.

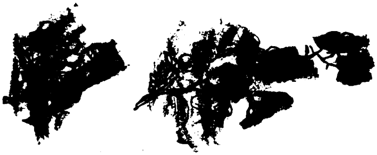
INCREASING AND DIMINISHING BY 4.

1. How many balls are 4 balls and 4 balls?

2. If we take away 4 marbles from 8 marbles, how many marbles will be left?

3. 5 sheep and 4 sheep are how many sheep? 5 and 4 are how many? 4 and 5 are how many?

4. If we take away 4 horses from 9 horses, how many horses will be left? 4 from 9 leaves how many? 5 from 9 leaves how many?



5. How many flowers are 6 flowers and 4 flowers? 6 and 4 are how many? How many are 4 and 6?

6. 4 roses from 10 roses leaves how many roses? 6 from 10 leaves how many? 4 from 10 leaves how many?

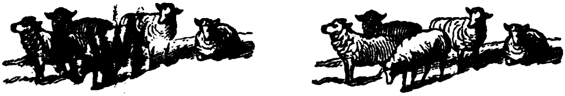
7. How many balls are 7 balls and 4 balls? 7 and 4 are how many? 4 and 7 are how many?

8. If we take 4 marbles from 11 marbles, how many marbles will be left? 4 from 11 leaves how many? 7 from 11 leaves how many?

9. Jane has eight pears, and Julia has four pears; how many pears have both? 8 and 4 are how many? 4 and 8 are how many? 4 and 5 and 2 are how many?

LESSON X.

INCREASING AND DIMINISHING BY 5.



1. How many sheep are 5 sheep and 5 sheep ?

2. If we take 5 sheep from 10 sheep, how many sheep will be left ? 5 from 10 leaves how many ?

3. There are 5 pears on one branch and 6 pears on the other; how many pears on both branches?

4. If we take 5 pears from 11 pears, how many pears will be left ? 6 from 11 leaves how many?

5. How many are 5 and 7? How many are 7 and 5 ?

6. If we take 5 lilies from 12 lilies, how many lilies will be left ? 5 from 12 leaves how many ? 7 from 12 leaves how many ?

7. How many acorns are 5 acorns and 8 acorns ? 5 and 8 are how many ? How many are 8 and 5 ?

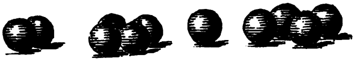
8. 5 from 13 leaves how many ? 8 from 13 leaves how many ?

9. Five and nine, how many ? 5 from 14, how many ?

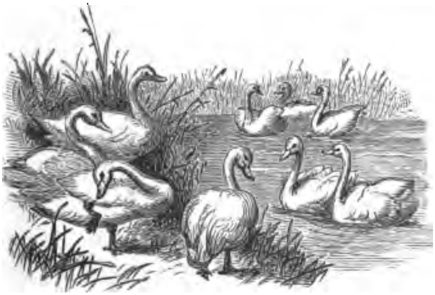
LESSON XI.

EXERCISES.

1. How many are nine cherries and three cherries? 9 and 3 are how many? 3 and 9 are how many?

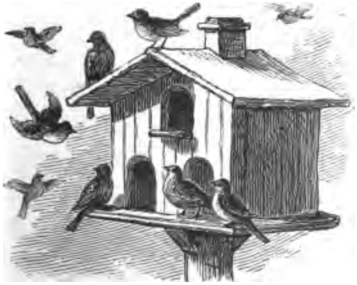


2. If we take 9 balls from 11 balls, how many balls are left? 9 from 11 leaves how many? 2 from 11 leaves how many? 9 books from 11 books how many?



3. Here is a flock of swans; 4 are on land, and 5 on the water: how many in all? How many are five and four?

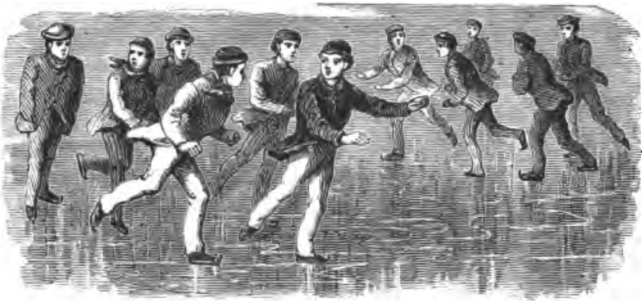
4. 4 from 9 leaves how many? 5 from 9 leaves how many?



5. How many birds are on the roof of the bird-house? How many are flying in the air? How many are on the shelf? How many are there in all? 2, 4 and 3 are how many? If the birds on the shelf fly away, how many will be left? 3 from 9 leaves how many?

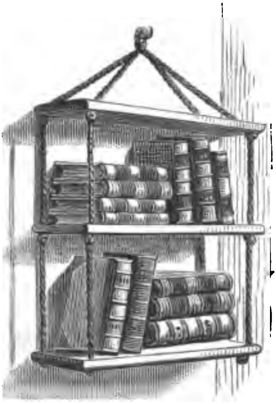
LESSON XII.

EXERCISES.



1. How many boys are skating towards the right? How many towards the left? How many in all?

2. If five leave the ice, how many will be left skating? 6 from 11 leaves how many? 5 from 11, how many?



3. Here is a book-rack containing books; some are standing up, and some are lying down. How many are standing on the lower shelf? How many are lying on the lower shelf? How many books are there altogether on the lower shelf?

4. How many books are standing on the upper shelf? How many are lying down? How many books are there altogether on the upper shelf?

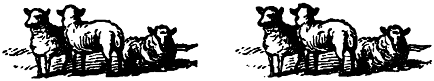
5. How many more are lying down on both shelves *than there are* standing?

LESSON XIII.

EXERCISES IN NUMBERS.

1. Two acorns and two acorns are how many acorns? How many lemons are 2 lemons and 2 lemons? How many acorns are 2 times 2 acorns? How many lemons are 2 times 2 lemons? How many are 2 times 2?

2. Two apples from four apples, leaves how many apples? 2 pears from 4 pears, leaves how many pears? How many times 2 apples are 4 apples? How many times 2 pears in 4 pears? 2 in 4 how many times?



3. How many sheep are 3 sheep and 3 sheep? How many sheep are 2 times 3 sheep? 3 times 2 sheep?

4. How many times 2 eggs, are there in 6 eggs? How many times 2 boys, in 6 boys? How many times 2 in 6? 3 times 2 are how many?

5. How many marbles are four marbles and four marbles? How many marbles are 2 times 4 marbles? How many are 2 times 4?

6. How many times 2 boats are there in 8 boats? How many times is 2 contained in 8?

7. If there are 2 bunches of acorns, and each bunch contains 5 acorns; how many acorns are there in both? How many are 2 times 5 acorns? How many are 2 times 5?

LESSON XIV.

WRITING HIGHER NUMBERS BY MEANS OF FIGURES.

Numbers from ninety-nine to one-thousand are written by three figures. The figure on the right, as we have already learned, stands for units, the second figure from the right stands for tens, the third figure stands for hundreds.

If there are no units, the figure on the right is 0.

If there are no tens, the second figure from the right is 0.

We will now write the numbers from one hundred to one hundred and twenty by means of figures:

101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120.

Two hundred is written by putting 2 in the place of hundreds, 0 in the place of tens, and 0 in the place of units; thus, 200.

Write two hundred and one; two hundred and two; two hundred and three; write two hundred and twelve.

Write three hundred; four hundred; five hundred; six hundred; seven hundred; eight hundred; nine hundred; two hundred and twenty-five.

Write nine hundred and nine; nine hundred and ninety nine; six hundred and four.

One thousand is written thus, 1000.

One thousand, one hundred, one ten, and one unit, as one number, are written thus, 1111; and the whole number is read one thousand one hundred and eleven.

Write in figures one thousand two hundred fifteen; one thousand and five; and one thousand and ten.

LESSON XV.

WRITING NUMBERS BY LETTERS.

We have learned two methods of writing numbers, one by *words*, and another by *figures*.

We will now learn a third method; the lessons in this book are numbered by this method.

In this method we use seven letters, I, V, X, L, C, D, M.

The following table shows how numbers are expressed by these seven letters.

TABLE.

1	I.	21	XXI.
2	II.	22	XXII.
3	III.	23	XXIII.
4	IV.	24	XXIV.
5	V.	25	XXV.
6	VI.	26	XXVI.
7	VII.	27	XXVII.
8	VIII.	28	XXVIII.
9	IX.	29	XXIX.
10	X.	30	XXX.
11	XI.	40	XL.
12	XII.	50	L.
13	XIII.	60	LX.
14	XIV.	70	LXX.
15	XV.	80	LXXX.
16	XVI.	90	XC.
17	XVII.	100	C.
18	XVIII.	500	D.
19	XIX.	1000	M.
20	XX.	10000	$\overline{\mathrm{M}}$.

NOTATION AND NUMERATION.

We have three methods of expressing numbers.

1. By words—called the **Word Method**; as one, two, three, four, etc.

2. By figures—called the **Arabic Method**; as 1, 2, 3, 4, etc.

3. By letters—called the **Roman Method**; as I, II, III, IV, etc.

Write all the numbers to fifteen, by each of the three methods.

The method of *writing* numbers is called **Notation.**

The method of *reading* written numbers is called **Numeration.**

RECAPITULATION AND DEFINITIONS.

1. A Unit is a single thing.

2. A Number is one or more things of the same kind.

3. A Figure is a character used to denote a number.

4. Notation is the method of writing numbers.

5. Numeration is the method of reading written numbers.

6. The Word Notation is the method of writing numbers by means of words.

7. The Arabic Notation is the method of writing numbers by means of figures.

8. The Roman Notation is the method of writing numbers by means of letters.

9. The ten figures taken separately are called **digits.** The naught is also called **cipher** or **zero,** and when considered by itself, has *no value.* The other figures are called **significant,** because each has a value.

10. Arithmetic is the science of numbers, and the art of computing by them.

TEST QUESTIONS.

What is a unit? What is a number? Write five numbers; write seven numbers. How many units in each? In how many ways can we express numbers? What is the first method? What is it called? What is the second? What is it called? What is the third? What is it called? What is notation? What is numeration? What is Arithmetic?

ORDERS OF UNITS.

11. How many fingers and thumbs have you? Write the number by a word.

Will any *one* of the ten figures express this number?

What is the greatest number that one figure will express?

One book is a single thing—a unit. If we make a bundle of ten books this *bundle* is a single thing, and is, therefore, a unit. But these units are not alike. One unit is a *single book;* the other is a *single bundle* of ten books. The single book is called a *unit of the first order;* the single bundle of ten books is called a *unit of the second order.*

How many single books make the single bundle?

How many units of the first order in one unit of the second order?

If we make ten bundles with ten books in each bundle and then place the ten bundles together, making one large bundle, the large bundle will also be *one thing*—a unit. How many small bundles in the large bundle?

This last mentioned unit is a unit of the third order. How many units of the second order make one unit of the third order?

Write the figure one, and at the left of it write another figure one. The first one is a unit of the first order. The second one is a unit of the second order.

How many units of the first order make one unit of the second order?

Write another figure one at the left of the second. This last one expresses a unit of the third order.

How many units of the second order make one unit of the third order?

Write another figure one at the left of the last. This is a unit of the fourth order. How many units of the third order make one unit of the fourth order?

Now write, without assistance, a unit of the fifth order and a unit of the sixth order.

Write with one figure two units of the first order; two of the second; two of the third; two of the fourth; two of the fifth; and two of the sixth.

NOTE.—*Always remember that one order of figures occupies but one place, and the largest number of any one order is nine.*

Write in figures, as one number, three units of the first order, four units of the second order, two units of the third order, five units of the fourth order, one unit of the fifth order, and nine units of the sixth order.

These numbers are *integers.*

12. An Integer is a whole number.

13. Units of the First Order either stand alone, or occupy the right-hand place.

14. Units of the Second Order occupy the second place from the right.

15. Units of the Third Order occupy the third place.

You may now tell what place units of the fourth order occupy; units of the fifth order, sixth order, seventh order, eighth order, ninth order, tenth order.

Units of the second order may be expressed without a unit of the first order, by putting a cipher in the place of the unit of the first order. Thus, 10.

The orders of units are *indicated* by the *relative positions of the figures.*

Units of any order may be written without expressing the units of other orders by putting ciphers in the place of the other units. Thus, two units of the third order are written 200; two units of the first order and four units of the fifth order are written thus, 40002, ciphers taking the place of the absent units.

Write two tens. What number have you written?

Write three tens, and at the right of it two units. What number have you now?

Write three units of the fourth order, and in the same line one unit of the third order, five units of the second order, and nine units of the first. What number do they express?

Units of the First Order express *single things,* and are called simply *units.*

Units of the Second Order express *collections of ten single things,* and are therefore called *tens.*

Units of the Third Order express *collections of ten tens,* or *one hundred,* and are therefore called *hundreds.*

Units of the Fourth Order express collections of *ten hundreds* or one *thousand,* and are therefore called *thousands,* as shown in the following

NUMERATION TABLE.

Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
2	1	3	5	4	6	9	7	8	0	7	8	4	5	3

Note to Teachers.—Minds are not all formed in the same mould. The teacher will find an unlimited variety. and must be ready to vary his methods of teaching accordingly.

SIMPLE AND LOCAL VALUES.

Another method of presenting this subject is the following. It is inserted to meet the wants of those who fail to comprehend the first method.

16. Figures have two values, called a **Simple Value** and a **Local Value.**

The **Simple Value of a Figure** is its value when standing alone, or when used as the right hand figure of a number.

The **Local Value of a Figure** is its value arising from the place in which it stands. When 2 stands alone or at the right hand, it denotes 2 units; when it stands in the second place from the right, it denotes 2 *tens*, as in 24; when it stands in the third place from the right, as in 234, it denotes 2 *hundreds*. The local values of figures increase from the right to the left by a **scale of tens.**

TEST QUESTIONS,

What do units of the first order express? Units of the second order? Third? Fourth? How many values have figures? What is the *simple* value of a figure? What is the *local* value? Write 2 so as to show its simple value. Write 2 so as to denote 2 tens; to denote 2 hundreds. How do the local values increase?

Places of *figures* and *orders of units* are counted from *right* to *left,* but numbers are read from *left* to *right.*

EXAMPLES IN NOTATION AND NUMERATION.

17. ***1.*** Numbers from one to nine inclusive are collections of simple units, and are expressed by a single figure.

2. Numbers from ten to ninety-nine inclusive are composed of tens and units. Thus, twenty-seven is composed of 2 tens and 7 units; forty-eight is composed of 4 tens and 8 units.

Write the following numbers by means of figures:

1. Twenty-five.
2. Forty-one.
3. Fifty-seven.
4. Eighty.
5. Ninety-one.
6. Thirty-two.
7. Fifty-eight.
8. Nineteen.
9. Twelve.
10. Seventy-six.
11. Forty-two.
12. Ninety-four.
13. Eighty-two.
14. Sixty-five.
15. Ninety-nine.

Read the following numbers:

84. 62. 94. 38. 27. 33. 76.
79. 41. 81. 54. 48. 87. 78.

3. Numbers from one hundred to nine hundred and ninety-nine inclusive are composed of hundreds, tens, and units. Thus, the number four hundred and sixty-five is composed of 4 *hundreds*, 6 *tens*, and 5 *units;* two hundred and three is composed of 2 *hundreds*, 0 *tens*, and 3 *units*.

EXAMPLES.

Write the following numbers by means of figures:

1. Two hundred and sixty-five.
2. Three hundred and ninety.
3. Seven hundred and eight.
4. Eight hundred and fifty-seven.
5. Nine hundred and eighty.
6. Four hundred and thirty-two.
7. Two hundred and six.
8. One hundred and ninety-nine.
9. Five hundred and seventy-three.
10. Six hundred and sixty-six.

4. To read a number of three figures, we name the number of hundreds and then read the tens and units, as though they were by themselves. Thus, 512 is read, five hundred and twelve; 874 is read, eight hundred and seventy-four; 209 is read, two hundred and nine.

Read the following numbers:

1. 713.
2. 806.
3. 200.
4. 817.
5. 728.
6. 827.
7. 495.
8. 888.
9. 232.
10. 450.
11. 527.
12. 932.
13. 642.
14. 404.
15. 546.
16. 634.
17. 978.
18. 769.
19. 384.
20. 763.
21. 914.
22. 571.
23. 994.
24. 999.

18. Numbers are written by putting each order in it own place, and if any order is not mentioned, naugh must occupy its place.

Write the following numbers by means of figures:

1. Six thousand four hundred and twenty-one.
2. One thousand three hundred and five.
3. Two thousand and ninety-six.
4. Eight thousand and one.
5. Four thousand and nine hundred.
6. Sixty-one thousand three hundred and seven.
7. Twenty-three thousand seven hundred and five.
8. Two hundred and fifty-nine thousand.
9. Three hundred, forty-eight thousand and thirteen.

Read the following numbers, and name the number o units in each order:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
1423	2010	23705	67832	258013
(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
2567	1365	61307	57243	700230

PERIODS OF FIGURES.

19. Numbers containing more than three figures ar separated into periods of three figures each, beginning a the right. The left-hand period may contain less tha three figures.

The first period, counting from the right, is called th *period of units,* the second is called the *period of thou sands;* the third is the *period of millions,* and so on, a shown in the following table:

Periods . . .	trillions,			billions,			millions,			thousands,			units.		
	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of
Number . . .	2	3	1,	8	7	6,	4	1	5,	3	0	0,	2	1	0,

The number written above is read, 231 *trillions,* 876 *billions,* 415 *millions,* 300 *thousands,* 210.

The table may be extended at pleasure; the units of the succeeding periods are *quadrillions, quintillions, sextillions,* etc.

Every period except the left-hand one must be *complete,* that is, it must contain three digits, but one or all these digits may all be ciphers.

Divide 146742684 36173 into periods. Tell how many figures in each period, and read the number.

DEFINITION.

20. A **Rule** is a brief direction for performing work.

RULE FOR NOTATION.

Begin at the left and write the figures of each period in their proper order, filling all vacant places with ciphers.

RULE FOR NUMERATION.

I. Begin at the right and separate the number into periods of three figures each, until you reach the left-hand period, which may have one, two, or three figures.

II. Begin at the left and read each period as if it stood alone, naming each period as you read its last figure.

Write, point off, and read the following numbers:

2345
14800
21576
743209
825364
421121
103043
7271856
234517
100200
89587346
6129456013
9078645327
12769853412
874218654214

TEST QUESTIONS.

How many periods in the last number? Name the periods in this number. How many figures in each period? Give the rule for notation. Give the rule for numeration. Which period may have less than three figures?

CLASSIFICATION OF NUMBERS.

Count five. In this manner of counting, you mention the numbers without a thought of any object. Numbers used in this way are called **abstract numbers.**

Count the number of scholars in this class; the number of maps on the wall; the number of books on my desk; the number of panes of glass in the window. Numbers used in connection with the objects counted, indicating the *number* of *objects,* are called **denominate** or **concrete.**

DEFINITIONS.

21. An **Abstract Number** is one whose unit is not named; as *three, five, seven.*

22. A **Denominate** or **Concrete Number** is one whose unit is named; as *three girls, five pounds seven pennies, eight pencils.*

Name five abstract numbers. Name five denominate numbers. Write four abstract numbers. Write five denominate numbers. What is an abstract number? What is a denominate number

23. An **Integer or Integral Number** is one which expresses one or more entire things.

24. The **Unit of any number** is one of the collection which constitutes the number.

25. Similar or **Like Numbers** are those that have the same kind of unit; as *eight days* and *ten days, two yards seven feet,* and *six yards eleven feet.*

26. Unlike Numbers are those that have different kinds of units; as *eight horses* and *five cows, six pencils and four knives, two feet* and *three days.*

REVIEW QUESTIONS.

What is a unit? Write a unit. What is a number? Write two numbers. What is arithmetic? What is notation? What is numeration? Name the three methods of notation. The Arabic notation employs how many figures? Write them. What are these figures called taken separately? What is an abstract number? Give two examples. What is a concrete, or denominate number? Give three examples. What is an integer? What are like numbers? Give two examples. What are unlike numbers? Give two examples. What are the first four orders of units? Write four units of the third order. How many units of any order make one unit of the next higher order? What is the greatest number that can be expressed by one figure? What is the greatest number that can be expressed by two figures? What by three figures? When there are four figures in a number, of what orders is it composed? Give an example and name the *order* of each figure. What do units of the first order denote? What do units of the second order denote? What do units of the third order denote? What is the simple value of a figure? What is the local value of a figure? Give all the general principles of notation and numeration. Give the rule for notation. Give the rule for numeration. How are numbers expressed in the Roman notation? What are figures? Name all the orders to trillions, beginning with units. Name the first four periods. Why is the second order called tens? Why third called hundreds? Why fourth called thousands?

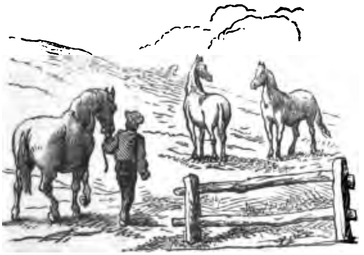
FUNDAMENTAL OPERATIONS.

ADDITION.

1. Mary has a *doll*, and her mother gives her another *doll;* how many *dolls* has she then?

Write the number of dolls that Mary has.

How many are 1 and 1? How many are 1 and 1 and 1? How many are 1 and 1 and 1 and 1?



2. If a farmer has 2 *horses* in the pasture and his neighbor puts in 1 more; how many *horses* will there be in the pasture? How many are 2 and 1? 3 and 1? 4 and 1? 5 and 1? 6 and 1? 7 and 1? 8 and 1? 9 and 1? 10 and 1? 12 and 1? 17 and 1?

Write the number of horses in the picture.

3. How many *dimes* are 3 *dimes* and 2 *dimes?* How many are 2, and 1, and 1? How many are 3, 1, and 1? How many are 4, 1, and 1? How many are 5, 1, and 1? How many are 6, 1, and 1?

4. How many *cents* are 4 *cents,* 2 *cents,* and 1 *cent?* How many are 3, 2, and 1? How many are 2, 3, and 1?

5. How many *men* are three *men,* two *men,* and two *men?* How many are 3, 2 and 2?

5. How many *marbles* are 4 *marbles,* 3 *marbles,* and 2 *marbles?* How many are 3, 4 and 2?

6. The word *bridge* has 6 *letters,* the word *man* has 3 *letters,* and the word *up* has 2 *letters;* how many *letters* in the three words? How many in the first two words?

The operation of finding how many *dolls* Mary has, how many horses there are in the pasture, etc., is called *Addition,* and the number thus found is called the *Sum* or *Amount.*

DEFINITIONS.

27. The sum of two or more numbers is a number which contains as many units as all the numbers taken together.

28. Addition is the operation of finding the sum of two or more numbers.

The numbers to be added must be similar, that is, they must have the same unit. Three days and two days can be added, because they have the same unit, one day; but three days and two yards cannot be added, because they have not the same unit.

What is the sum or amount of two or more numbers? What is addition? What are similar or like numbers?

SIGNS.

In the examples given above, the word *and* is used to denote the *addition;* we generally denote it by this sign +, which is called **plus**, and when it is used between numbers it shows that they are to be added; thus $6+3+2$ are 11, means that the sum of *six* and *three* and *two* is equal to *eleven.*

In place of the word *are*, the sign = is used. It is called the *sign* of *equality*, and is read *equals* or *equal to;* thus, $6 + 4 + 8 = 18$, is read, *six* plus *four* plus *eight* equals *eighteen.*

What is the sign of addition? Make it. What does it denote? What is the sign of equality? Make it.

29. The sign of equality placed between numbers or combinations of numbers, shows that those at the left hand are equal to those at the right.

The entire expression is called an **equation**; thus, $6 + 3 = 9$, $7 - 2 = 5$, $8 \times 3 \div 2 = 14 - 6 + 4$, are equations.

30. An Arithmetical Equation is the expression of equality between numbers or combinations of numbers.

What is an equation? Write an equation.

EXERCISES FOR ORAL WORK.

1+1 = ?
2+1 = ?
3+1 = ?
4+1 = ?
6+1 = ?

2+2 = ?
3+2 = ?
4+2 = ?
5+2 = ?
6+2 = ?

3+3 = ?
4+3 = ?
5+3 = ?
6+3 = ?
7+3 = ?

4+4 = ?
5+4 = ?
6+4 = ?
7+4 = ?
8+4 = ?

31. Make and learn the following

ADDITION TABLE.

2+ 0= 2	3+ 0= 3	4+ 0= 4	5+ 0= 5
2+ 1= 3	3+ 1= 4	4+ 1= 5	5+ 1= 6
2+ 2= 4	3+ 2= 5	4+ 2= 6	5+ 2= 7
2+ 3= 5	3+ 3= 6	4+ 3= 7	5+ 3= 8
2+ 4= 6	3+ 4= 7	4+ 4= 8	5+ 4= 9
2+ 5= 7	3+ 5= 8	4+ 5= 9	5+ 5=10
2+ 6= 8	3+ 6= 9	4+ 6=10	5+ 6=11
2+ 7= 9	3+ 7=10	4+ 7=11	5+ 7=12
2+ 8=10	3+ 8=11	4+ 8=12	5+ 8=13
2+ 9=11	3+ 9=12	4+ 9=13	5+ 9=14
2+10=12	3+10=13	4+10=14	5+10=15
6+ 0= 6	7+ 0= 7	8+ 0= 8	9+ 0= 9
6+ 1= 7	7+ 1= 8	8+ 1= 9	9+ 1=10
6+ 2= 8	7+ 2= 9	8+ 2=10	9+ 2=11
6+ 3= 9	7+ 3=10	8+ 3=11	9+ 3=12
6+ 4=10	7+ 4=11	8+ 4=12	9+ 4=13
6+ 5=11	7+ 5=12	8+ 5=13	9+ 5=14
6+ 6=12	7+ 6=13	8+ 6=14	9+ 6=15
6+ 7=13	7+ 7=14	8+ 7=15	9+ 7=16
6+ 8=14	7+ 8=15	8+ 8=16	9+ 8=17
6+ 9=15	7+ 9=16	8+ 9=17	9+ 9=18
6+10=16	7+10=17	8+10=18	9+10=19

Note.—The teacher will be amply repaid for thorough drill in all possible combinations of the digits. The pupil should be thoroughly master of the table. Frequent exercises are required to secure this and to break up the habit of counting, which is fatal to rapidity in addition.

EXAMPLES FOR ORAL WORK.

1. 2+7 = ?
2. 4+9 = ?
3. 8+6 = ?
4. 3+5 = ?
5. 9+5 = ?
6. 8+4 = ?
7. 8+9 = ?
8. 3+8 = ?
9. 9+2 = ?
10. 7+8 = ?
11. 6+9 = ?
12. 8+7 = ?

13. 3+4+2+5 = ?
14. 4+5+3+4 = ?
15. 7+3+2+5 = ?
16. 9+3+2+4 = ?
17. 1+3+4+3+2 = ?
18. 3+2+3+4+2+3 = ?

19. 4+3+2+3+1+6 = ? 3+8+5+4+6 = ?
20. 5+2+3+1+2+4+5 = ? 7+9+8+5 = ?
21. 8+5+4+2+3+1+7 = ? 9+3+6+4 = ?
22. 9+6+3+4+6+2+1 = ? 4+9+8+7 = ?

Note.—The signs of interrogation in these and all similar examples indicate that the second members are to be supplied by the pupil.

23. Count by twos from 2 to 50; thus, 2, 4, 6, 8, 10, etc.
24. Count by threes from 3 to 99; from 27 to 120.
25. Count by fours from 0 to 100; from 105 to 160.
26. Count by fives from 1 to 106; from 200 to 250.
27. Count by sixes from 0 to 108; from 212 to 242.
28. Count by sevens from 0 to 98; from 100 to 135.
29. Count by eights from 0 to 104; from 150 to 174.
30. Count by threes from 4 to 46; from 50 to 77.
31. Count by fives from 0 to 100; from 200 to 250.
32. Count by nines from 0 to 99; from 100 to 154.

32. Instead of writing numbers in a horizontal line with the sign between them, it is more convenient to write them in columns without any sign, the sum being written beneath. In the following examples, add from the bottom upward.

EXAMPLES FOR WRITTEN WORK.

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)
4	6	8	8	9	8	6	1	5
5	8	3	8	3	4	7	2	9
7	2	8	4	9	3	3	5	4
3	9	7	7	5	5	8	9	7
19								

In adding, name only the results of each addition: thus, Example (1), *three, ten, fifteen, nineteen;* Example (2), *nine, eleven, nineteen, twenty-five.*

Prove the work by adding from the top downward; if the same sum is obtained, the work is thought to be right.

Add and prove the following examples:

(*10*)	(*11.*)	(*12.*)	(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)	(*18.*)
7	9	7	8	5	8	5	7	8
8	8	3	4	7	1	5	6	3
1	4	6	9	4	6	3	5	2
5	3	5	3	3	9	2	9	5
4	9	2	2	9	4	9	4	7

(*19.*)	(*20.*)	(*21.*)	(*22.*)	(*23.*)	(*24.*)	(*25.*)	(*26.*)	(*27.*)
8	8	4	4	1	9	4	6	8
4	2	6	5	3	1	5	2	9
3	8	5	9	7	8	7	6	4
6	3	3	2	2	6	2	3	2
2	7	8	1	4	3	8	4	1
1	9	9	3	9	2	6	7	8

28. Find the sum of 3, 4, 8, 9, 7, 4, and 5.

29. Find the sum of 6, 3, 2, 1, 8, 9, 4, 5, and 9.

30. Find the sum of 9, 8, 6, 7, 2, 5, 8, 7, and 17.

31. Find the sum of 8, 7, 9, 6, 3, 5, 1, 4 and 5.

Simple numbers may be added by the following

RULE.

I. Write the numbers so that units of the same order shall stand in the same column.

II. Begin at the right, add each column, and write the sum, if less than ten, under the column.

III. When the sum of any column exceeds nine, set down the right-hand figure of the sum under the column, and add the number indicated by the left-hand figure or figures to the next column.

IV. Continue this operation till all the columns have been added; write the entire sum of the last column.

Proof.—*Add the numbers from the top downward; if the result is the same as the first sum, the work is presumed to be right.*

EXAMPLES.

	(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
	24	30	45	81	16	42
	32	23	33	72	12	33
	51	64	61	63	27	21
	70	72	70	54	38	48
Sum,	177					

(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
36	18	84	44	88	80
47	41	62	37	66	60
58	82	31	18	23	45
79	61	14	29	32	57

	(*13.*)	(*14.*)	(*15.*)	(*16.*)
	213	127 *days*	236 *quarts*	812
	416	213 *days*	72 *quarts*	174
	729	418 *days*	801 *quarts*	702
Sum,	1358	758 *days*		

In the last four examples, which answers are abstract? Which are concrete?

Can 127 *days* be added to 236 *quarts?* Why not?

EXAMPLES.

17. Find the sum of 125, 718, 64, 376, and 715.

18. Find the sum of 73 *years,* 172 *years,* 60 *years,* 812 *years,* 43 *years,* and 197 *years.*

19. Add 345 *quarts,* 117 *quarts,* 123 *quarts,* 885 *quarts,* 64 *quarts,* and 543 *quarts.*

20. Add 2,135 *pounds,* 8,126 *pounds,* 3,152 *pounds,* 8,176 *pounds,* and 364 *pounds.*

21. Find the sum of 77, 213, 315, 421, and 607.

Add the following groups of numbers:

22. 818, 328, 40, 671, 364, 484, and 793.

23. 15, 812, 75, 717, 645, 720, and 347.

24. 412 *days,* 817 *days,* 516 *days,* and 893 *days.*

ABBREVIATIONS.—In what follows, *ft.* stands for *feet; yds.* for *yards; lbs.* for *pounds; in.* for *inches; qts.* for *quarts;* and the sign $ placed before a number stands for *dollars.*

When dollars and cents are expressed, we first write the sign, then the number of dollars, then a point or period, and then the number of cents: thus, $25.75, read twenty-five dollars and seventy-five cents. If the number of cents to be written is less than ten, a cipher must be put in the tens place; thus, $16.05, read sixteen dollars and five cents. If cents alone are to be written, we first make the sign of dollars, then a 0, then the point, and then write the number of cents; thus, $0.16, read 16 cents. When dollars, cents, and mills are to be expressed, write the dollars and cents as above, and the mills at the right of the cents. Seven dollars, twenty-five cents and eight mills are written $7.258.

EXAMPLES.

Find the sum of

25. 512 *ft.*, 893 *ft.*, 911 *ft.*, 745 *ft.*, and 14 *ft.*
26. 482 *yds.*, 886 *yds.*, 924 *yds.*, 87 *yds.*, and 994 *yds.*
27. 864 *lbs.*, 342 *lbs.*, 182 *lbs.*, 94 *lbs.*, and 14 *lbs.*
28. 667 *in.*, 843 *in.*, 918 *in.*, 445 *in.*, and 887 *in.*
29. $818, $435, $88, $413, $867, $983, and $71.
30. 2,314, 12,107, 210,026, 78,784, and 68,547.
31. $2,308, $12,125, $41,410, and $18,876.
32. 1,280 *yds.*, 71,413 *yds.*, 47,489 *yds.*, and 9,297 *yds.*
33. 1,762 *lbs.*, 4,389 *lbs.*, 120,000 *lbs.*, and 172,794 *lbs.*
34. 842 *ft.*, 8,848 *ft.*, 27,796 *ft.*, 152,407 *ft.*, and 18 *ft.*
35. $64, $640, $6,832, $9,040, $118,920, and $8,734.
36. 614 *in.*, 3,260 *in.*, 89,705 *in.*, 8,884 *in.*, and 286 *in.*

DEFINITIONS.

33. A **Problem** is a question requiring a solution.

34. A **Solution** is the operation of finding the required answer.

PRACTICAL PROBLEMS FOR ORAL WORK.

35. *1.* Jane's father gave her 6 peaches and her mother gave her 4 more; how many peaches did both give her?

SOLUTION. 6 peaches + 4 peaches = 10 peaches. *Ans.*

2. Charles gave 10 cents for a Faber pencil No. 2, 5 cents for an Eagle pencil No. 2, and 8 cents for a Stoddard pencil; what did the three pencils cost him?

3. The head of a fish caught in Newark Bay was 6 inches long, its body 21 inches long, and its tail 7 inches long; how long was the fish?

4. Count by 7's from 2 to 79; from 5 to 96; from 4 to 102; from 9 to 51; from 11 to 74.

5. George solved 19 problems in the morning and 8 in the evening; how many did he solve in all?

FOR WRITTEN WORK.

6. A grocer has 3 hogsheads of sugar, of which the first weighs 957 *lbs.*, the second 1,023 *lbs.*, and the third 1,179 *lbs.*; what is the weight of them all?

EXPLANATION. — The weight of the whole is equal to the sum of the weights of all the parts. Hence, we set down the separate weights and add them.

ILLUSTRATION.

957 *lbs.*
1023 *lbs.*
1179 *lbs.*
Ans. 3159 *lbs.*

7. A merchant bought 4 pieces of cloth for $129, 6 pieces of silk for $312, and 97 pieces of muslin for $873; what did he pay for them all? *Ans.* $1,314.

8. A gentleman bought a pair of horses for $650, a set of harness for $190, and a carriage for $955; what did they all cost?

9. A merchant bought a horse for 112 dollars; after keeping him a short time, he sold him, and gained 25 dollars; how much did he receive for the horse?

10. The mail route from Albany to New York is 144 miles, from New York to Philadelphia 90 miles, from

Philadelphia to Baltimore 98 miles, and from Baltimore to Washington City 38 miles; what is the distance from Albany to Washington?

FOR ORAL WORK.

36. *1.* A man bought a firkin of butter for $9, a keg of syrup for $6, and 5 bushels of wheat for $7; how much did he give for the whole?

2. A boy gave to one of his companions 7 apples, to another 6, and to a third 8; how many apples did he give away? $7+6+8=?$

3. A farmer bought a cow for $30, a sheep for $20, and a calf for $10; how much did he give for the whole?

4. In a young peach orchard Jane found 27 peaches on one tree, on another 10, on another 8, and on another 5; how many peaches did she find?

5. A lady bought a muff for $25, a boa for $15, and a pair of gaiters for $10; how much did she pay for the whole? $\$25+\$15+\$10=?$

FOR WRITTEN WORK.

1. A grocer sold 289 pounds of sugar for $28, ten barrels of flour for $108, and a quantity of pork for $879; what did he get for the whole?

2. A person pays $950 for a lot of ground, on which he builds a house costing $5,430, a barn costing $986, and then sells the whole so as to gain $914; what was his selling price?

3. A farmer raises 673 bushels of wheat, 1,489 bushels of corn, 67 bushels of barley, and 1,682 bushels of oats; how many bushels of grain does he raise in all?

4. A farmer sells his stock of cattle as follows: for his oxen he gets $883, for his cows $1,279, for his calves $413, and for his horses $980; what does he get for them all?

5. A gentleman builds a house; his lot costs him $1,254, the carpenter work costs $4,320, the masonry $2,110, the painting and papering $1,187, and the miscellaneous expenses amount to $1,277; what is the cost of the whole?

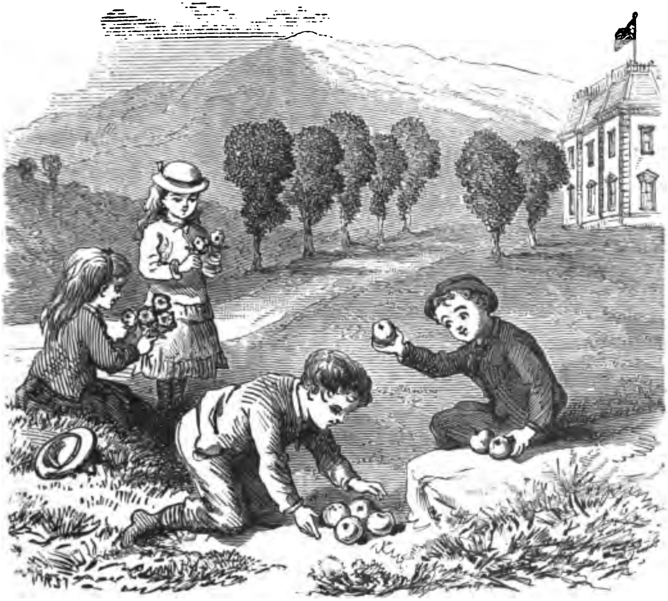
6. The distance from Boston to Springfield is 99 miles, from Springfield to Albany 102 miles, from Albany to Rochester 226 miles, from Rochester to Buffalo 65 miles, and from Buffalo to Chicago 518 miles; how far is it from Boston to Chicago by this route?

7. A manufacturer paid $8,820 for rent, $17,780 for material, $47,885 for labor, and then sold his goods so as to clear $11,827; what was the amount of his sales?

8. A speculator bought a house and lot for 1,964 dollars, expended 384 dollars in repairing and refitting the property, paid taxes and insurance amounting to 56 dollars, and then sold them so as to gain 396 dollars; what did he get for the property?

TEST QUESTIONS.

What is addition? What is the answer in addition called? What is the sign of addition? What does it mean? Make the sign of equality. Why is it called sign of equality? Write an example in which there is the sign of equality, and show how it is used. Give the rule for writing numbers in addition. Give the rule for adding and writing the results. How do you prove addition? What is an equation? What are the members of an equation? What numbers can be added together? Make the sign for dollars. How many orders of units or places do cents occupy? What sign is used between dollars and cents? How are dollars, cents, and mills written for adding? How many places do cents and mills occupy?



SUBTRACTION.

1. One of the boys in the picture has 4 *apples*, the other boy has 3 *apples;* how many more *apples* has the first boy than the second?

2. One of the girls has 4 *roses*, the other has 2 *roses;* how many more *roses* has one girl than the other?

3. On one side of the walk there are 5 *trees*, on the other side 2 *trees;* how many more *trees* on one side than on the other?

4. On one side of the house you can see 6 *windows*, on the other side 2 *windows;* how many more can you see on one side than on the other?

5. A man had 12 *cows,* and sold 3 of them; how many had he left?

In these five examples we are required to find *how much greater one number is than another.* The number thus found is the **Difference** *between the two numbers,* and the process of finding it is called **Subtraction.** The difference is also called a **Remainder.**

DEFINITIONS.

37. The **Difference,** or **Remainder,** is a number which shows how much greater one of two numbers is than the other.

38. **Subtraction** is the operation of finding the difference between two numbers.

In these examples, and in all examples of subtraction in Arithmetic, the greater number is called the **Minuend.** The less number is called the **Subtrahend.**

39. The **Minuend** is one of two numbers from which the other is to be subtracted.

40. The **Subtrahend** is the number to be subtracted.

What is meant by the difference between two numbers? What is subtraction? What is the minuend? What is the subtrahend? In each of the following examples tell which is the minuend and which the subtrahend.

Read and work the following

EXAMPLES.

1. 5 less 4 = 1.
2. 7 less 6 = how many?
3. 4 less 1 = how many?
4. 5 less 3 = how many?
5. 4 less 3 = how many?
6. 6 less 1 = how many?
7. 6 less 4 = how many?
8. 5 less 1 = how many?
9. 4 less 2 = how many?
10. 3 less 1 = how many?
11. 8 less 3 are how many? 8 less 4? 8 less 5?

Instead of the word *less* between two numbers whose difference is required, this sign — is used. It is called the **Minus Sign, or Sign of Subtraction.**

41. Minus denotes *less*, and when placed between two numbers it shows that the second is to be subtracted from the first. Thus, 5 — 3 shows that 3 is to be taken from 5.

42. The **Parenthesis,** ( ), is used to show that the expression enclosed by it is to be treated as a single number. Thus, 8 — (3 + 2) shows that the sum of 3 and 2 is to be subtracted from 8.

Read and work the following

EXAMPLES.

1. 6—1 = 5.
2. 4—1 = ?
3. 3—2 = ?
4. 5—2 = ?
5. 3—1 = ?
6. 8—4 = ?
7. 5—3 = ?
8. 4—3 = ?
9. 2—1 = ?
10. 6—2 = ?
11. 5—4 = ?
12. 7—2 = ?
13. 4—2 = ?
14. 6—3 = ?
15. 5—1 = ?
16. 6—4 = ?
17. 6—5 = ?
18. 9—6 = ?

19. 10—3 = how many? 10—5? 10—7? 10—8?
20. 12—1 = how many? 12—2? 12—3? 12—6?
21. How many are 13—1? 13—2? 13—3? 13—4?
22. How many are 14—1? 14—2? 14—3? 14—4?

Write the sign of subtraction. What is it called? What does it mean? When used between two numbers what does it show?

Read 8 — 3 = 5, and tell which is the minuend, which the subtrahend, and which the remainder?

Can you always tell, if the sign of subtraction is used, which is the minuend, and which is the subtrahend? How?

43. Write and learn the following

SUBTRACTION TABLE.

1 from	2 from	3 from	4 from	5 from
2 leaves 1	3 leaves 1	4 leaves 1	5 leaves 1	6 leaves 1
3 " 2	4 " 2	5 " 2	6 " 2	7 " 2
4 " 3	5 " 3	6 " 3	7 " 3	8 " 3
5 " 4	6 " 4	7 " 4	8 " 4	9 " 4
6 " 5	7 " 5	8 " 5	9 " 5	10 " 5
7 " 6	8 " 6	9 " 6	10 " 6	11 " 6
8 " 7	9 " 7	10 " 7	11 " 7	12 " 7
9 " 8	10 " 8	11 " 8	12 " 8	13 " 8
10 " 9	11 " 9	12 " 9	13 " 9	14 " 9
6 from	7 from	8 from	9 from	10 from
7 leaves 1	8 leaves 1	9 leaves 1	10 leaves 1	11 leaves 1
8 " 2	9 " 2	10 " 2	11 " 2	12 " 2
9 " 3	10 " 3	11 " 3	12 " 3	13 " 3
10 " 4	11 " 4	12 " 4	13 " 4	14 " 4
11 " 5	12 " 5	13 " 5	14 " 5	15 " 5
12 " 6	13 " 6	14 " 6	15 " 6	16 " 6
13 " 7	14 " 7	15 " 7	16 " 7	17 " 7
14 " 8	15 " 8	16 " 8	17 " 8	18 " 8
15 " 9	16 " 9	17 " 9	18 " 9	19 " 9

NOTE.—This table should be repeated until the scholar becomes familiar with it. Change the form thus: 2−1=1, etc. Again, change thus: What number taken from 2 leaves 1? etc.

The minuend, the subtrahend, and the remainder must be *similar*, or *like numbers*.

EXAMPLES.

ORAL WORK.

1. $7 - 4 = ?$
2. $8 - 3 = ?$
3. $9 - 6 = ?$
4. $12 - 9 = ?$
5. $11 - 4 = ?$
6. $6 - ? = 2.$
7. $5 - ? = 4.$
8. $10 - ? = 3.$
9. $12 - ? = 5.$
10. $13 - ? = 8.$
11. $12 - 3 = ?$
12. $14 - 5 = ?$
13. $15 - 6 = ?$
14. $16 - 7 = ?$
15. $15 - 8 = ?$
16. $15 - ? = 10.$
17. $13 - ? = 4.$
18. $14 - ? = 8.$
19. $12 - ? = 5.$
20. $15 - ? = 9.$

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21. $5 + 6 - 3 = ?$
22. $7 + 8 - 4 = ?$
23. $9 + 3 - 5 = ?$
24. $12+2+3-4 = ?$
25. $9+7+6-5 = ?$
26. $20 - (2 + 4) = ?$
27. $15 - (3 + 5) = ?$
28. $18-(1+2+3) = ?$
29. $20 - (3+4+5) = ?$
30. $19 - (7+6+1) = ?$
31. $12 - 2 - 2 - 2 = ?$
32. $15 - 3 - 2 - 4 = ?$
33. $20 - 2 - 5 - 3 = ?$
34. $18 - 5 - 4 - 3 = ?$
35. $20 - 6 - 5 - 3 = ?$
36. $18 + 3 -(6- 3)= ?$

37. 20 diminished by 5 = ? 15 diminished by 5 ? 10 diminished by 5 ? 5 diminished by 5 ?

38. In the same manner name the numbers from 52 to 0, diminishing each by 2 ; from 78 to 52 ; from 100 to 78.

39. What are the numbers from 86 to 2, when each is diminished by 6 ? from 122 to 8 ?

44. Instead of writing numbers for subtraction in a horizontal line with minus between, it is generally more convenient to write the subtrahend under the minuend, placing the remainder beneath; thus,

12 *Minuend.*
7 *Subtrahend.*
5 *Remainder.*

In this manner work the following

EXAMPLES.

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
16	18	12	10	9	8	12	20	16	21
9	4	5	7	4	5	3	9	7	5
7									

To *prove the work*, add the remainder to the subtrahend, and if the sum is equal to the minuend the work is presumed to be right.

Perform and prove the following

EXAMPLES.

	(*11.*)	(*12.*)	(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)	(*18.*)	(*19.*)
From	9	7	11	15	8	13	17	10	14
Subtract	2	3	7	6	4	5	9	6	8

	(*20.*)	(*21.*)	(*22.*)	(*23.*)	(*24.*)	(*25.*)	(*26.*)
From	14 *yds.*	12 *lbs.*	19 *ft.*	$13	16 *in.*	10 *lbs.*	15 *qts.*
Subtract	9 *yds.*	6 *lbs.*	10 *ft.*	$4	7 *in.*	3 *lbs.*	9 *qts.*

	(*27.*)	(*28.*)	(*29.*)	(*30.*)	(*31.*)	(*32.*)	(*33.*)	(*34.*)
From	8	10 *lbs.*	9 *yds.*	12 *ft.*	14 *in.*	17	$15	33 *days.*
Take	4	6 *lbs.*	2 *yds.*	4 *ft.*	8 *in.*	8	$9	21 *days.*

Can 9 *pounds* be subtracted from 12 *yards*? Why not?

EXERCISES FOR WRITTEN WORK.

45. *When the figures of the subtrahend are equal to or less than the corresponding figures of the minuend.*

EXPLANATION.—We write the subtrahend under the minuend so that units stand under units, tens under tens, and hundreds under hundreds. We begin at the right and subtract 2 *units* from 5 *units* and write the remainder, 3 *units*, beneath. We then subtract 3 *tens* from 9 *tens* and write the result, 6 *tens*, in the column of tens. Then we subtract 5 *hundreds* from 7 *hundreds*, and write the difference in the column of hundreds.

ILLUSTRATION.

From	795
Subtract	532
Remainder,	263

PROOF.

Add the subtrahend and remainder, and if the sum equals the minuend the work is right.

532	Subtrahend.
263	Remainder.
795	= the Minuend.

SECOND METHOD OF PROOF.

Subtract the remainder from the minuend, and if the result equals the subtrahend the work is right.

795	Minuend.
263	Remainder.
532	Subtrahend.

In the same manner work and prove the following

EXAMPLES.

	(1.)	*(2.)*	*(3.)*	*(4.)*	*(5.)*	*(6.)*
From	87	76	95	69	82	98
Subtract	34	35	63	24	51	72
Remainder,						

	(7.)	*(8.)*	*(9.)*	*(10.)*	*(11.)*	*(12.)*
From	54 *yds.*	68 *yds.*	$27	69 *lbs.*	96 *in.*	79 *ft.*
Subtract	22 *yds.*	35 *yds.*	$16	27 *lbs.*	24 *in.*	27 *ft.*

Construct an example in subtraction having 3 orders of units, in which the numbers are abstract.

Construct two examples, in which the numbers are concrete, each having three orders of units.

Which is the minuend? Which is the subtrahend? What is the remainder in each example?

How do you prove subtraction?

46. *When any figure in the subtrahend is greater than the corresponding figure in the minuend.*

If we subtract 4 from 7 we get 3. If we add 5 to both minuend and subtrahend, and then subtract, we get the same remainder 3. Thus,

From	7	From	$7 + 5 = 12$
Take	4	Take	$4 + 5 = 9$
Rem.	3	Rem.	3

Also, add 9 to both minuend and subtrahend. Thus,

From	$7 + 9 = 16$
Take	$4 + 9 = 13$
Rem.	3

From these illustrations we see that if the same number be added to both minuend and subtrahend, the remainder is not changed.

From	5482
Subtract	2625
Rem.	2857

EXPLANATION.—Since 2 *units* are less than 5 *units*, we add 1 *ten* to 2 *units*, making 12 *units*, and subtract 5 *units* from 12 *units*, leaving 7 *units*, which we write beneath. To balance the 1 *ten* added to the minuend we add 1 *ten* to the subtrahend, making 3 *tens*, which we subtract from 8 *tens*, leaving 5 *tens*; this we write below. Since 4 *hundreds* are less than 6 *hundreds*, we add 10 *hun-*

dreds **to 4 *hundreds*, making 14 *hundreds*, from which subtract 6 *hundreds*, and write the difference, 8 *hundreds*, beneath. To balance 10 *hundreds*, equal to 1 *thousand*, added to the minuend, we add 1 *thousand* to the subtrahend, making 3 *thousands*, which we subtract from 5 *thousands* and write the remainder, 2 *thousands*, beneath, making our entire remainder 2,857.**

From these illustrations and principles we deduce the following

RULE.

I. Write the less number under the greater so that units of the same order shall stand in the same column.

II. Beginning at the right hand, subtract each figure in the lower line from the one above it and set the remainder in the line below.

III. If a figure in the lower line is greater than the one above it, increase the latter by 10, perform the subtraction, and then add 1 to the next figure in the lower line.

EXAMPLES FOR WRITTEN WORK.

	(1.)	*(2.)*	*(3.)*	*(4.)*	*(5.)*	*(6.)*
From	61	73	64	82	94	58
Subtract	29	48	27	76	78	39
Rem.						

	(7.)	*(8.)*	*(9.)*	*(10.)*	*(11.)*
From	578	964	887	843	765
Subtract	343	352	324	232	234
Rem.					

	(12.)	*(13.)*	*(14.)*	*(15.)*	*(16.)*
From	897 *lbs.*	415 *ft.*	679 *yds.*	$813	589 *in.*
Subtract	534 *lbs.*	203 *ft.*	234 *yds.*	$261	98 *in.*
Rem.					

	(17.)	*(18.)*	*(19.)*	*(20.)*
From	2,843	$5,946	2,003	8,800 *ft.*
Subtract	1,678	$1,389	976	2,088 *ft.*
Rem.				

(21.)	*(22.)*	*(23.)*	*(24.)*	*(25.)*
4760	3695	87231	67087	12048
1986	1863	1009	40000	8034

26. From 287 subtract 114.

27. From 994 subtract 363.

28. From 10,841 subtract 3,009.

29. From $12,560 subtract $4,885.

30. From 115,440 *ft.* subtract 19,359 *ft.*

31. From 1,310,844 subtract 337,775.

32. From $4,478 take $989.

33. From 77,475 *yds.* take 10,994 *yds.*

34. Find the difference between $785 and $323.

35. Find the difference between 12,843 *yds.* and 2,318 *yds.*

36. Find the difference between 711,711 and 82,082.

37. What is the difference between 5,858 *ft.* and 949 *ft.*?

38. How much does 244,887 exceed 108,104?

39. 2,478 + 1,236 — (2,562 — 1,893) = ?

TEST QUESTIONS.

What is subtraction? What is the minuend? What is the subtrahend? What is the difference, or remainder? How are the numbers written for work in subtraction? When the figures of the subtrahend are equal to or less than the corresponding figures of the minuend, how do you subtract? When a figure in the subtrahend is greater than its corresponding figure in the minuend, how do you subtract? How do you prove subtraction? Give the rule for subtraction. What is a problem? What is the sign of subtraction? How is the parenthesis used in examples of subtraction?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

47. *1.* A man having $10, paid $6 for a pair of boots; how many dollars had he left?

Solution.—He had the difference between $10 and $4, which is $6.

2. A boy had 20 marbles, and gave away 7 of them; how many had he left? 20 — 7 = ?

3. Mary is 15 years old, and Jane is 9; how long before Jane will be 15? 15 — 9 = ?

4. Two boys have together 20 *cts.*; if one has 12 *cts.*, how many has the other? 20 — 12 = ?

5. James bought a book for 20 *cts.*, and paid a 25 *ct.* piece; how much change should he receive?

6. Charles has 50 *cts.*, he pays 10 *cts.* for a pencil and 5 *cts.* for a rubber; how many cents has he left?

FOR WRITTEN WORK.

7. A merchant bought 750 *yds.* of cloth and sold 468 *yds.* of the same; how many *yds.* had he left?

8. From a flock containing 718 sheep 432 were sold; how many remained? 718 — 432 = ?

9. A man bought a pair of horses for $788, and sold them again for $629; how much did he lose?

10. A merchant bought a stock of goods for $1,887 and sold the same for $2,143; how much did he gain?

11. A man's income is $6,000 per annum and his expenses are $4,125; how much does he save?

FOR ORAL WORK.

48. *1.* A man 50 years old has a son 20 years old; how much older is the father than the son?

2. A man having 30 miles to travel in 2 days, goes 18 miles the first day; how far must he go the second day?

3. A woman has $21 and spends $12; how much money has she left? $21 — $11 = ?

4. John weighs 60 *lbs.* and Mary 50 *lbs.*; how much heavier is John than Mary? 60 *lbs.* — 50 *lbs.* = ?

5. A man had $100, $80 of which was in the bank and the rest in his pocket; how much in his pocket?

6. A man purchased a farm for $10,000, and paid cash $6,000, how much remained unpaid?

FOR WRITTEN WORK.

7. A merchant sells goods for $17,480 and gains by the sale $4,894; what did they cost him?

8. A farmer had 497 sheep, but he sold at one time 113 and at another time 98; how many had he left?

9. A merchant begins business with $18,413; the first year he loses $800 and the second year he gains $976; what is he then worth?

10. A man has an income of $2,500 per annum, and he spends $750 for rent, $1,200 for living expenses, and the remainder he saves; how much does he save per year?

11. A drover bought 711 sheep of one farmer, 310 sheep of another, and then sold 462; how many had he left? 711 + 310 — 462 = ?

FOR ORAL WORK.

49. *1.* A boy having 15 *cts.* spent 5 *cts.* for a pencil and 8 *cts.* for a sponge; how much money had he left?

2. A boy has $30 in the bank; he draws out $7 at one time and $9 at another; how much remains in the bank?

3. A boy bought sugar for 10 *cts.* and eggs for 12 *cts.*, and gave the clerk 25 *cts.*; how much change should he receive? 25 — (10 + 12) = ?

4. A boy who had 10 marbles bought 15 more, and he then lost 12; how many had he left?

5. A man bought a horse for $60, a harness for $20, a wagon for $30; he afterward sold them all for $100; how much did he lose? 60 + 40 + 30 — (100 + 20) = ?

FOR WRITTEN WORK.

6. From a regiment of 847 men 143 were discharged and 273 were killed in battle; how many remained?

7. A trader commences business with a capital of $3,245; the first year he gains $422, the second year he gains $500; the third year he loses $792, and the fourth year he loses $117; how much is he then worth?

8. A man had $13,850 in the bank, but drew out at one time $1,872, at another time $3,814, and at a third time $4,811; how much had he then in bank?

9. A man gave to his four sons $3,780; to the first he gave $1,490, to the second he gave $1,109, to the third he gave $675, and to the fourth he gave the remainder; how much did he give the fourth?

10. Four men bought a tract of land, for which they paid $8,419; the first paid $3,815, the second paid $2,140, the third paid $1,480; what did the fourth pay?

FOR ORAL WORK.

50. *1.* An orchard contained 50 trees, 10 of which were peach trees, 5 pear trees, 8 plum trees, and the rest were apple trees; how many apple trees in the orchard?

2. I bought a coat for $20, a vest for $8, pants for $10, and I paid a $50 bill; how much did I receive in return? $\$50 - (\$10 + \$8 + \$12) = ?$

3. A dish contained 60 peaches, Jane took 12, Susan 10, Mary 13, and John 15; how many were left in the dish?

4. Six men bought a horse for $150; the first gave $50, the second $30, the third $25, the fourth $18, and the fifth $10; how much did the sixth give?

5. A farmer bought a horse for $100, and paid $15 for keeping him; he let him enough to receive $25 and then sold him for $90; did he gain or lose by the bargain? How much?

FOR WRITTEN WORK.

6. A man gave to his four sons $5,880; to the first he gave $2,360, to the second he gave $2,109, to the third he gave $805, and to the fourth he gave the remainder; how much did he give the fourth?

7. A householder sold two houses; for the first, which cost $3500, he received $4760; for the second, which cost $3735, he received $5000; on which of the houses did he make the greater gain, and how much?

8. A person borrowed of his neighbor at one time $355, at another time $637, and $403 at another time; he paid him $977; how much did he then owe him?

9. I have a yearly income of $10,000. I pay $275 for office rent, $220 for fuel, $35 to the doctor, and $3675 for all my other expenses; how much have I left at the end of the year?

10. A man pays $300 for 100 sheep, $95 for a pair of oxen, $60 for a horse, and $125 for a chaise; he gives 100 bushels of wheat worth $125, a cow worth $25, a colt worth $40, and pays the rest in cash; how much money does he pay?

11. If the subtrahend be 750 and the remainder 964, what is the minuend?

12. If the minuend be 60,402 and the remainder 29,475, what is the subtrahend?

13. The difference of two numbers is 607 and the greater number is 1,005; what is the less number?

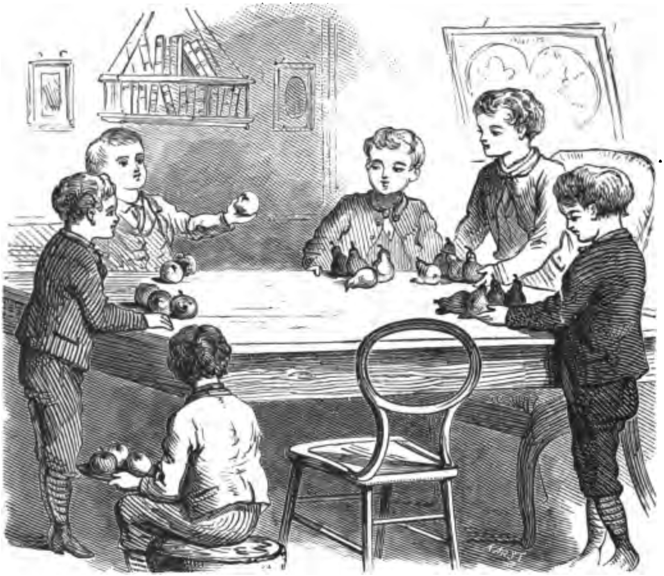
14. 7,963 + 54,923 + 27,984 — 64,937 = ?

15. 22,786 — (10,342 + 5,684) = ?

16. A man worth $18,000 left $4,287 to his elder son, $3,754 to his younger son, $3,219 to his daughter, and the remainder to his wife; what was the wife's portion?

REVIEW QUESTIONS.

What is a unit? What is a number? What is an abstract number? What is a concrete number? What is a simple number? What is a compound number? Define notation. Define numeration. Give all the methods of notation. Write seven thousand two hundred and fifty-one by means of the Arabic notation. Write five hundred and seventy-eight by means of the Roman notation. Numerate 7,852,643,827,462, and read the number. What is addition? What numbers can be added together? Give the rule for addition. How do you prove addition? Make the sign of addition. Of equality. Make the sign of dollars. When dollars and cents are written, how many orders of units are occupied by cents? What sign is put between dollars and cents? When dollars, cents, and mills are written, how many orders of units do mills occupy? What is subtraction? Define minuend. Define subtrahend. Define remainder. Make the sign of subtraction, name it, and tell how it is used. Give the rule for subtraction. Work the following example: 178,462—(6,895 + 18,754). When the difference and the greater of two numbers are given, how do you find the less? Suppose the sum of three numbers and two of them are given, how will you find the third? Construct a problem illustrating each of the above questions.



MULTIPLICATION.

51. *1.* In this picture there are two groups of boys and 3 boys in each group; how many boys are there in the picture?

SOLUTION.—Since there are 3 boys in one group, in 2 groups there are 2 times three boys, 3 + 3 = 6, 3 taken twice = 6.

2. In one group, each boy has 3 apples; how many apples have the 3 boys?

SOLUTION.—Since each boy has 3 apples, 3 boys have 3 times 3 or 9 apples, 3 + 3 + 3 = 9, or 3 taken 3 times = 9.

3. In the other group, each boy has 4 pears; how many pears have the 3 boys.

SOLUTION.—Since each boy has 4 pears, 3 boys will have 3 times four or 12 pears, 4+4+4 = 12, or 4 taken 3 times = 12.

4. How many trees in 4 rows, if there are 5 trees in each row? 4 times 5 trees are how many trees?

SOLUTION. 5+5+5+5 = 20, or 5 taken 4 times = 20.

5. How many hands have 6 boys?

SOLUTION. 2+2+2+2+2+2 = 12, or 2 taken 6 times = 12.

6. How many feet have 6 horses?

SOLUTION. 4+4+4+4+4+4 = 24, or 4 taken 6 times = 24.

7. John's father gave him 6 five-cent pieces; how many cents did the father give John?

SOLUTION. 5+5+5+5+5+5 = 30, or 5 taken 6 times = 30.

8. Bought 3 *lbs.* of sugar at 10 *cts.* a pound; how much did the sugar cost? 3 times 10 *cts.* = ?

SOLUTION. 10+10+10 = 30, 10 taken 3 times = 30.

In the first example we find the sum of 2 threes, or 3 taken twice. In the second we find the sum of 3 threes, or 3 units taken 3 times. In the fourth we find the sum of 3 fours, or 4 taken 3 times, and so on.

In the first example, how many times is 3 taken? What is the result? In the fifth example, how many times is the number 2 taken? In the sixth example, how many times is 4 taken? What number is taken 6 times in the seventh example? How many times is 10 taken in the eighth?

52. The operation of taking a number a certain number of times is called **Multiplication.**

The number to be taken is called the **Multiplicand,** and the number which shows how many times the multiplicand is taken is called the **Multiplier.**

DEFINITIONS.

53. **Multiplication** is the operation of taking one number as many times as there are units in the other.

54. The **Multiplicand** is the number to be taken or multiplied.

55. The **Multiplier** is the number which shows how many times the multiplicand is to be taken, or what part of it is to be taken.

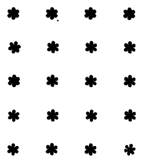
56. The **Product** is the result of the multiplication.

What is multiplication? What is the multiplicand? What is the multiplier? In each of the eight examples on pages 60 and 61, tell which number is the multiplicand? Which is the multiplier? In Example 1st, 6 is the product; in Example 2d, 9 is the product; in Example 3d, 12 is the product. Tell what is the product in each of the other examples.

57. The multiplicand and multiplier are called **Factors** of the product.

58. The following is the **Sign of Multiplication,** ×. When placed between two numbers it is read *multiplied by;* thus, 3 × 2 is read *3 multiplied by 2.*

The value of the product does not depend on the order in which the factors are taken. Thus, 4 times 5 is the same as 5 times 4, as shown in the diagram; for, if we take the stars by rows, we have 4 stars taken 5 times; if we take them by columns, we have 5 stars taken 4 times; in either case there are 20 stars. The multiplier, however, must always be considered an abstract number. The multiplicand and product are like numbers, and may be abstract or concrete.



59. The **elements of multiplication** are given in the following table, called the

MULTIPLICATION TABLE.

Once			2 times			3 times			4 times		
1	is	1	1	are	2	1	are	3	1	are	4
2	"	2	2	"	4	2	"	6	2	"	8
3	"	3	3	"	6	3	"	9	3	"	12
4	"	4	4	"	8	4	"	12	4	"	16
5	"	5	5	"	10	5	"	15	5	"	20
6	"	6	6	"	12	6	"	18	6	"	24
7	"	7	7	"	14	7	"	21	7	"	28
8	"	8	8	"	16	8	"	24	8	"	32
9	"	9	9	"	18	9	"	27	9	"	36
5 times			6 times			7 times			8 times		
1	are	5	1	are	6	1	are	7	1	are	8
2	"	10	2	"	12	2	"	14	2	"	16
3	"	15	3	"	18	3	"	21	3	"	24
4	"	20	4	"	24	4	"	28	4	"	32
5	"	25	5	"	30	5	"	35	5	"	40
6	"	30	6	"	36	6	"	42	6	"	48
7	"	35	7	"	42	7	"	49	7	"	56
8	"	40	8	"	48	8	"	56	8	"	64
9	"	45	9	"	54	9	"	63	9	"	72
9 times			10 times			11 times			12 times		
1	are	9	1	are	10	1	are	11	1	are	12
2	"	18	2	"	20	2	"	22	2	"	24
3	"	27	3	"	30	3	"	33	3	"	36
4	"	36	4	"	40	4	"	44	4	"	48
5	"	45	5	"	50	5	"	55	5	"	60
6	"	54	6	"	60	6	"	66	6	"	72
7	"	63	7	"	70	7	"	77	7	"	84
8	"	72	8	"	80	8	"	88	8	"	96
9	"	81	9	"	90	9	"	99	9	"	108

NOTE.—The Multiplication Table should be *perfectly committed to memory.*

EXERCISES FOR MENTAL WORK.

1. 5 × 4 = 20.
2. 7 × 8 = ?
3. 6 × 7 = ?
4. 8 × 6 = ?
5. 9 × 4 = ?
6. 3 × 7 = ?
7. 6 × 9 = ?
8. 12 × 6 = ?
9. 7 × 9 = ?
10. 8 × 7 = ?
11. 4 × 9 = ?
12. 12 × 4 = ?
13. 9 × 9 = ?
14. 4 × 7 = ?
15. 9 × 8 = ?
16. 12 × 7 = ?
17. 10 × 8 = ?
18. 11 × 9 = ?
19. 12 × 11 = ?
20. 6 × 8 = ?
21. 8 × 9 = ?

What are the factors in the first example above? What in the fifth? What in the tenth? What in the twentieth? Name the factors of 6; of 10; of 14; and of 15.

Supply the missing factor in each of the following:

1. 7 × =21.
2. × 4=36.
3. × 9=54.
4. 12 × =60.
5. × 6=42
6. 7 × = 56.
7. 11 × =121.
8. × 9=108.
9. × 7= 63.
10. 12 × = 96.
11. 12 × =60.
12. × 6=30.
13. × 9=72.
14. 7 × =28.
15. × 8=32.

If 4 marbles be taken 5 times, which is the abstract number? Which the concrete? Which is the multiplicand? Which the multiplier? What is the product? Is the product abstract or concrete?

EXERCISES FOR ORAL WORK.

60. *1.* If 1 *lb.* of rice costs 11 *cts.*, how much will 4 *lbs.* cost? 11 *cts.* × 4 = how many *cts.*?

2. 7 boys receive 8 *cts.* each; how many cents do all receive? 8 *cts.* × 7 = how many *cts.*?

3. What will 9 lemons cost, at 9 *cts.* each?

4. How much can a boy earn in 8 weeks, if he earn $7 each week? $7 × 8 = how many dollars?

5. A man walks 6 miles a day for 12 days; how far does he go? 6 × 8 = how many? 6 × 9 = ?

6. There are 3 feet in 1 yard; how many feet in 6 yards? How many in 7 yards? In 8 yards? 10 yds.?

7. Bought 12 *yds.* of cloth at $5 a yard; how much did it cost? How much did 10 *yds.* cost? 12 *yds.*?

8. What will 4 pairs of boots cost at $8 a pair?

9. In 1 peck there are 8 quarts; how many quarts in 9 pecks? How many in 10 pecks? In 12 pecks?

10. There are 12 inches in 1 foot; how many inches in 11 feet? In 12 *ft.*? In 9 *ft.*? In 8 *ft.*? In 7 *ft.*?

EXERCISES FOR WRITTEN WORK.

1. $50 × 4 = ?
2. 19 *cts.* × 6 = ?
3. 16 *ft.* × 9 = ?
4. 32 *in.* × 3 = ?
5. 8 *apples* × 7 = ?
6. 12 *marbles* × 3 = ?
7. 16 *horses* × 4 = ?
8. 9 *sheep* × 2 = ?
9. 8 *miles* × 5 = ?
10. 7 *pounds* × 6 = ?

Instead of writing in a horizontal line, with the sign × between the factors, it is more convenient to write the multiplier under the multiplicand.

61. *When the multiplier consists of but one figure.*

Multiply 328 by 7.

ILLUSTRATION.

Multiplicand,	328
Multiplier,	7
Product,	2,296

EXPLANATION.—Multiplying 8 by 7, we have 56, that is, 5 *tens* and 6 *units;* we set down the 6 *units* and carry forward the 5 *tens* to the product of 2 tens multiplied by 7. Multiplying 2 *tens* by 7, we have 14 *tens,* which, increased by the 5 *tens* brought forward, gives 19 *tens,* or 1 *hundred* and 9 *tens;* we set down the 9 *tens* and carry forward the 1 *hundred.* Multiplying 3 *hundreds* by 7, we have 21 *hundreds*, which, increased by the 1 *hundred* brought forward, gives 22 *hundreds*, or 2 *thousands* and 2 *hundreds;* this we set down. The required product is 2,296.

NOTE.—The operation of multiplying may be abbreviated, as explained in addition, by omitting the names of the figures, and simply naming the results of the successive multiplications.

RULE.

Begin at the right and multiply each figure of the multiplicand by the multiplier, setting down and carrying as in addition.

EXAMPLES.

Perform the following multiplications:

	(1.)	*(2.)*	*(3.)*	*(4.)*
Multiplicand,	134	318	256	808
Multiplier,	3	4	2	4
Product,	402			

	(5.)	*(6.)*	*(7.)*	*(8.)*
Multiplicand,	318	476	1234	4137
Multiplier,	5	7	8	9
Product,				

	(9.)	*(10.)*	*(11.)*	*(12.)*
Multiplicand,	417 *ft.*	1843 *in.*	$894	693 *lbs.*
Multiplier,	6	7	5	9
Product,	2502 *ft.*	*in.*	$4470	*lbs.*

(13.)	*(14.)*	*(15.)*	*(16.)*
3145	43214 *lbs.*	81342	$5486
8	7	6	9
25160	*lbs.*	488052	$

What is the rule when the multiplier consists of but one figure?

62. *When the multiplier contains any number of figures.*

EXAMPLE.

1. Multiply 458 by 346.

EXPLANATION.—Here we write the numbers so that units of the same order stand in the same column. We then multiply 458 by 6, which gives 2748. We next multiply 458 by 4 *tens*, and set the first figure in the column of tens. We then multiply 458 by 3 *hundreds*, and place the first figure in the column of hundreds. Adding the partial products thus obtained, we find for the total product 158,468.

ILLUSTRATION.

Multiplicand,	458
Multiplier,	346
Partial Products.	2748
	1832
	1374
Product,	158468

From this illustration and explanation we deduce the following

RULE.

I. Write the multiplier under the multiplicand, so that units of the same order shall stand in the same column.

II. Begin at the right and multiply the multiplicand by each figure of the multiplier, writing the first figure of each partial product under the corresponding figure of the multiplier.

III. Find the sum of the partial products.

PROOF.—*Multiply the multiplier by the multiplicand, and if the second product equals the first the work is presumed to be right.*

EXAMPLES FOR WRITTEN WORK.

(*1.*)	*Proof.*	(*2.*)	*Proof.*
78	64	86	67
64	78	67	86
4992	4992	5762	5762

Multiply

3. 7,406 by 36.
4. 3,421 by 48.
5. 8,413 by 75.
6. 719 by 183.
7. 743 by 345.
8. 828 by 712.
9. 294 by 252.
10. 813 by 712.
11. 576784 by 64.
12. 596875 by 144.
13. 46123101 by 72.
14. 6185720 by 132.
15. 718328 by 96.
16. 679534 by 9185.
17. 86972 by 1208.
18. 1055054 by 279.

Multiply

19. 6,431 by 27.
20. 2,782 by 28.
21. 9,346 by 54.
22. 1,243 by 126.
23. 873 by 284.
24. 1,349 by 236.
25. 2,157 by 317.
26. 3,184 by 196.
27. 792 by 215.
28. 349 by 318.
29. 92 by 47.
30. 1,894 by 23.
31. 757 by 132.
32. 2,416 by 99.
33. 1,308 by 102.
34. 3,047 by 205.

What is the rule when the multiplier contains any number of figures? What is the method of proof?

63. *When one or both factors have ciphers on the right.*

Every cipher that we annex to a whole number moves each of its digits one place to the left; but this is the same as multiplying by 10.

Hence, to multiply a whole number by 10, we annex *one* cipher to the multiplicand; to multiply by 100, we annex *two* ciphers; to multiply by 1,000, we annex *three* ciphers; and so on. Thus, 75 × 10 = 750; 34 × 100 = 3,400; 87 × 1000 = 87,000.

If the significant figure of the multiplier followed by ciphers is greater than 1, we first multiply by the significant figure or figures, and then annex the ciphers.

Multiply 317 by 300.

EXPLANATION.—We multiply 317 by 3, which gives 951, and to this we annex two ciphers, as shown in the illustration.

ILLUSTRATION.

$$\begin{array}{r} 317 \\ 300 \\ \hline 95100 \end{array}$$

EXAMPLES.

Multiply

1. 318 by 20.
2. 914 by 900.
3. 8,143 by 500.
4. 4,175 by 80.
5. 874 *yds.* by 300.
6. $841 by 70.
7. 8,888 by 3,700.

Multiply

8. 406 by 400.
9. 516 by 800.
10. 217 by 2000.
11. 429 by 400.
12. $27 by 601.
13. 927 by 1200.
14. 561 by 2,050.

If both factors terminate in ciphers, multiply the significant figures, and to the result annex as many ciphers as there are at the right of both factors.

Multiply 8900 by 9000.

EXPLANATION.—Here we multiply 89 by 9, which gives 801, and to the result we annex five ciphers as shown in the illustration.

ILLUSTRATION.

$$\begin{array}{r} 8900 \\ 9000 \\ \hline 80100000 \end{array}$$

EXAMPLES.

15. Multiply 870 by 300.
16. Multiply 41,900 by 90.
17. 2500 by 500.
18. 5600 by 2000.

Multiply

19. 8,100 by 7,000.
20. 7,300 by 50.
21. 3,460 by 80.
22. $20,370 by 70.
23. 5,150 *yds.* by 600.

Multiply

24. 7,050 by 200.
25. 400 by 300.
26. 7100 by 50.
27. 3912 by 600.
28. 4200 *ft.* by 30.

One cipher annexed to a number moves each digit how many places to the left? This is the same as multiplying by what number? How do you multiply by 10? How by 100? How by 1,000? When the multiplier is a significant figure followed by one or more ciphers how do you multiply?

COMPOSITE NUMBERS.

64. A **composite** number is one that can be separated into other integral factors than itself and one. Thus, 6 is a composite number, because it can be separated into the factors 2 and 3.

Note.—Scholars should be carefully taught to distinguish between *factors* of a number and *parts* of a number. Any number is the sum of its parts, but the product of its factors.

Exercises for Oral Work.

Separate into two factors each of the following numbers:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
10,	15,	21,	14,	22,	25,	26,	32,	40,	42.

Separate into three factors each of the following numbers:

(*11.*)	(*12.*)	(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)	(*18.*)	(*19.*)	(*20.*)
8,	12,	16,	24,	27,	30,	36,	48,	56,	45.

To multiply by a composite number, we may multiply by each of its factors in succession. Thus, to multiply 118 by 24, we may multiply 118 by 6, which gives 708, and then multiply the result by 4, which gives 2,832; this is the required product.

In the same manner multiply the following

EXAMPLES.

21. Multiply 78 by 48, (6 × 8).
22. Multiply 96 by 108, (9 × 12).
23. Multiply 413 by 56, (7 × 8).
24. Multiply 88 by 3 × 7, (21).
25. Multiply 546 by 8 × 12, (96).
26. Multiply 8,342 by 7 × 12, (84).
27. Multiply 8 × 5 by 7 × 6.
28. Multiply 15 × 18 by 16 × 12.

The product of more than two factors is called a **Continued Product.** Thus, in example 27, the number 1,680 is the continued product of 8, 5, 7, and 6.

What is a composite number? How do you multiply by the factors of a composite number? What is a continued product?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

65. *1.* What will 7 hundred pounds of sugar cost at $9 a hundred? $9 × 7 = how many dollars?

2. 4 quarters make 1 yard; how many quarters in 8 yards? 4 times 8 equals how many?

3. If a man earn $7 a week, how much will he earn in 10 weeks? How much in 12 weeks?

4. How many yards of cloth in 7 pieces, each piece containing 10 yards? How many in 8 pieces?

5. What will 5 barrels of flour cost at $8 a barrel?

FOR WRITTEN WORK.

6. What will 38 barrels of flour cost at $13 a barrel?

Ans. $494.

7. What is the cost of 675 *lbs.* of cheese at 14 cents a pound? At 12 cents a pound? At 11 cents a pound?

8. A farmer sold 211 bushels of potatoes at 74 cents a bushel; how much did he receive?

9. What is the cost of 786 quarts of milk at 9 cents a quart? What at 10 cents a quart?

10. If $1 will buy 21 tickets, how much will $37 buy?

FOR ORAL WORK.

66. *1.* In one yard there are 3 feet; how many feet in 12 yards? In 9 yards? In 7 yards? In 11 yards?

2. How many feet in 6 yards and 2 feet? In 7 yards and 2 feet? In 6 yards and 1 foot?

3. If one quarter of a yard of beaver cloth costs $2, what will 1 yard cost? What will 2 yards cost?

4. If 4 bushels of wheat make one barrel of flour, how many bushels will be required to make 9 barrels?

5. A gentleman bought 10 yards of silk at $2 a yard, and 6 pairs of stockings at 50 *cts.* a pair; how much should he pay for the goods?

FOR WRITTEN WORK.

6. If $1 will buy 4 *lbs.* of butter, how many pounds will $82 buy? How many *lbs.* will $37 buy?

7. If $1 will buy 8 *lbs.* of sugar, how many pounds will $17 buy? How many will $13 buy?

8. There are 3,600 seconds in 1 hour; how many seconds are there in 24 hours, or 1 day?

9. How many seconds are there in two days?

10. If a chest of tea contains 64 *lbs.* and each pound is worth 70 cents, what will be the value of 18 chests?

11. A drover bought 74 head of cattle at $82 a head, and sold the lot for $7,500; how much did he make?

FOR ORAL WORK.

67. *1.* 10 decimeters make 1 meter; how many decimeters make 6 meters?

2. 10 centimeters make one decimeter; how many centimeters in 8 decimeters?

3. 10 millimeters make one centimeter; how many millimeters make 9 centimeters?

4. On a chess-board there are eight rows of squares and eight squares in each row; how many squares are there on the board?

5. Two men start from the same place and travel in opposite directions; one travels 2 miles an hour, the other travels 3 miles an hour; how far apart will they be at the end of 5 hours?

6. Two men start from the same place and travel the same way; one travels 2 miles an hour and the other 3 miles an hour; how far apart will they be at the end of 8 hours?

7. Bought 3 meters of linen at $2 a meter, 7 meters of silk at $3 a meter, 5 meters of ribbon for $4, some crape for $2, and gave the merchant 4 ten-dollar bills; how much change should I receive back?

FOR WRITTEN WORK.

8. A farmer has 3 flocks of sheep, numbering respectively 50, 60, and 75 head, and each sheep yields 4 *lbs.* of wool; what is the value of his wool crop when wool is worth 36 cents a pound?

9. Two couriers travel toward each other, the first at the rate of 35 miles and the second at the rate of 42 miles a day; at the end of 9 days they are separated by 411 miles; how far apart were they at first?

10. A person bought 30 *yds.* of muslin at 20 *cts.* a yard, 4 *yds.* of silk at $1.75 a yard, and 14 books at 77 cents each; what was the amount of his bill?

11. What is the difference between 118 times 327 and 211 times 82?

12. (2134 + 506) × (1800 − 500) = ?

13. (32 × 6) + (48 × 9) − (17 × 4 − 3) + 160 = ?

14. ($2478 − $1032) × (2041 + 453) × 9 − 7 = ?

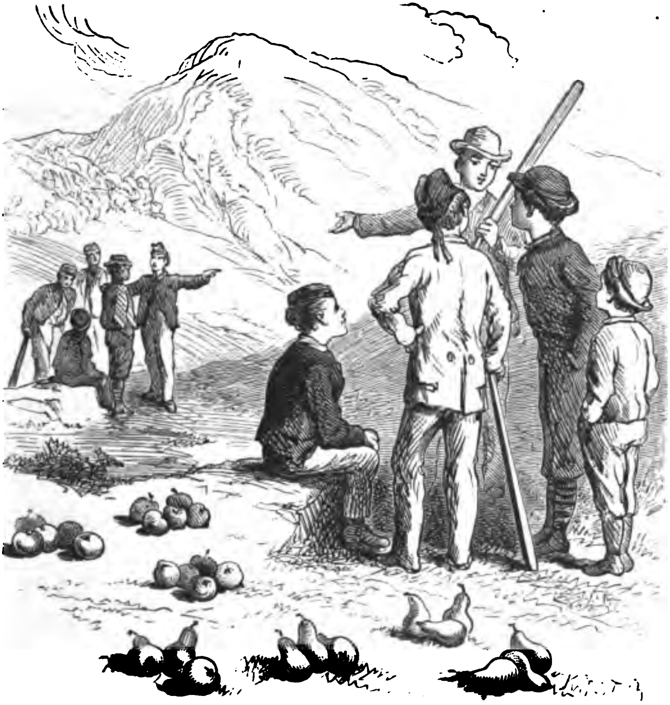
15. What is the sum of 512 times 384, and 81 times 611?

16. (3042 *yds.* − 2106 *yds.* + 218 *yds.*) × (354 − 214) × (27 + 3) = ?

17. 2304 + 38 − (640 − 84) × 16 − 6.

REVIEW QUESTIONS.

Recite the multiplication table. What is multiplication? What is the multiplicand? What is the multiplier? What is the product? Make the sign of multiplication; tell how it is used and how it is read. Which of the two factors is always considered abstract? In the operation of multiplication how are the multiplier and multiplicand written when the sign is not used? Give the rule when the multiplier consists of but one figure. Give the rule when the multiplier consists of more than one figure. What is meant by the factors of the product? What is a composite number? How do you multiply by the factors of a composite number? What is the process when ciphers occur on the right of one or both factors. What is the use of the parentheses in Examples 12, 13, 14 and 16.



DIVISION.

68. How many boys in this picture?

Into how many groups are they divided? How many boys in each group?

1. If 10 boys are divided into two equal groups, how many boys are there in each group?

2. If 15 apples are separated into 3 equal piles, how many apples in each pile?

3. If 12 pears are divided among four boys, how many pears will each boy receive?

4. If 20 cents will buy 5 oranges, how many cents will buy 1 orange?

5. If a man earns $18 in 6 days, how many dollars does he earn in 1 day?

6. In 2 days there are 48 hours, how many hours in 1 day?

7. If 3 yards of silk cost $12, what will 1 yard cost?

8. There are 24 boys in 2 classes, with an equal number in each, how many boys in each class?

9. Paid 30 cents for 5 oranges, how many cents did 1 orange cost?

10. How many barrels of apples can be bought for $40, if each barrel costs $5.

In the first example we are required to find one of two equal parts of ten. In the third we find the number of equal parts in 12, each of which contains 3 units.

Hence, in division, we aim at **one of two objects**; either to find the *number of units in each of the equal parts* of a given number, or the *number of equal parts* into which a given number is to be divided.

The number divided is called the **Dividend**. The number which shows into how many parts the dividend is divided is called the **Divisor**. That which shows how many times the divisor is contained in the dividend is called the **Quotient**.

DEFINITIONS.

69. Division is the operation of finding how many times one number is contained in another, or of finding one of the equal parts of a number.

70. The **Dividend** is the number to be divided.

71. The **Divisor** is the number by which the dividend is to be divided.

72. The **Quotient** is the result of the division, and shows how many times the divisor is contained in the dividend.

Examine carefully the 10 examples given, pages 75 and 76, and tell which is the *dividend* in each example, which the *divisor*, and what is the *quotient*.

SIGNS OF DIVISION.

73. There are *three methods* of indicating division.

1. By a *horizontal line* with a *point or period* above and below it; thus, ÷. This sign, when standing between two numbers, shows that the first is to be divided by the second; thus, 8 ÷ 2 is read 8 divided by 2.

2. By a *horizontal line* with the *dividend written above*, and the *divisor below;* thus, $\frac{8}{2}$, read 8 divided by 2.

3. By a *curved line* with the *divisor at the left*, and the *dividend at the right;* thus, 2) 8, read 8 divided by 2.

Write the expression 16 divided by 2, by each of the three methods.

Read the following examples:

12 ÷ 3 = 4. 48 ÷ 6 = 8. $\frac{72}{9} = 8$. 2)92 = 46.
24 ÷ 2 = 12. 84 ÷ 7 = 12. $\frac{63}{7} = 9$. 3)36 = 12.
35 ÷ 7 = 5. 100 ÷ 10 = 10. $\frac{54}{9} = 6$. 5)45 = 9.

Elements of division in which the divisors are graded from 1 to 12 are given in the following

DIVISION TABLE.

1 in			2 in			3 in			4 in		
2	2	times	4	2	times	6	2	times	8	2	times
3	3	“	6	3	“	9	3	“	12	3	“
4	4	“	8	4	“	12	4	“	16	4	“
5	5	“	10	5	“	15	5	“	20	5	“
6	6	“	12	6	“	18	6	“	24	6	“
7	7	“	14	7	“	21	7	“	28	7	“
8	8	“	16	8	“	24	8	“	32	8	“
9	9	“	18	9	“	27	9	“	36	9	“
5 in			6 in			7 in			8 in		
10	2	times	12	2	times	14	2	times	16	2	times
15	3	“	18	3	“	21	3	“	24	3	“
20	4	“	24	4	“	28	4	“	32	4	“
25	5	“	30	5	“	35	5	“	40	5	“
30	6	“	36	6	“	42	6	“	48	6	“
35	7	“	42	7	“	49	7	“	56	7	“
40	8	“	48	8	“	56	8	“	64	8	“
45	9	“	54	9	“	63	9	“	72	9	“
9 in			10 in			11 in			12 in		
18	2	times	20	2	times	22	2	times	24	2	times
27	3	“	30	3	“	33	3	“	36	3	“
36	4	“	40	4	“	44	4	“	48	4	“
45	5	“	50	5	“	55	5	“	60	5	“
54	6	“	60	6	“	66	6	“	72	6	“
63	7	“	70	7	“	77	7	“	84	7	“
72	8	“	80	8	“	88	8	“	96	8	“
81	9	“	90	9	“	99	9	“	108	9	“

74. One object of division is to divide a given number into equal parts.

A————————————B

Let AB be a line one foot long; if we divide it into two equal parts, each part is one-half of a foot.

Division may be expressed by writing the dividend above a horizontal line, and the divisor below; hence, 1 divided by 2 may be written $\frac{1}{2}$.

As a quotient, $\frac{1}{2}$ is read *one-half.*

If the same line is divided into 3 equal parts, we have $\frac{1}{3}$ (1 divided by 3), which as a quotient is read *one-third.*

If we divide it into 4 equal parts, we have $\frac{1}{4}$ (1 divided by 4), read as a quotient *one-fourth.*

NOTE.—The quotient of 2 by 3 may be written $\frac{2}{3}$; this is *one-third* of 2, or it is *two-thirds* of 1. The quotient of 3 by 7 may be written $\frac{3}{7}$; this is *one-seventh* of 3, or it is *three-sevenths* of 1. The expression $\frac{2}{3}$ is read *two-thirds;* $\frac{3}{7}$ is read *three-sevenths;* $\frac{4}{9}$ is read *four-ninths;* $\frac{11}{13}$ is read *eleven-thirteenths;* and so on. Expressions of the kind just explained are called *fractions.*

75. A Fraction is one or more equal parts of a unit.

Read the following fractions:

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{1}{10}$, $\frac{2}{5}$,
$\frac{2}{11}$, $\frac{3}{13}$, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{7}{18}$, $\frac{9}{19}$, $\frac{3}{20}$, $\frac{8}{21}$, $\frac{7}{8}$, $\frac{9}{40}$.

Write by means of figures:

One-half, One-third, One-fourth, Two-fifths, One-sixth, Three-sevenths, One-eighth, Two-elevenths, Five-ninths, Seven-fifteenths, Nine-elevenths, Two-thirds, Seventeen-twentieths, Eight-seventeenths, Five-ninths, Nineteen-thirtieths, Two-thirteenths, Twenty-fifteenths, and Ninety one-hundredths.

Supply the missing numbers in the following exercises:

42 ÷ = 6.
÷ 8 = 4.
56 ÷ 8 =
63 ÷ = 3.
÷ 5 = 7.
72 ÷ 12 =

81 ÷ = 9.
44 ÷ = 4.
÷ 6 = 9.
÷ 4 = 12.
60 ÷ = 12.
84 ÷ = 14.

72 ÷ = 9.
84 ÷ = 7.
÷ 8 = 12.
÷ 11 = 12.
÷ 7 = 9.
100 ÷ 10 =

The quotient of any number by 2 is one-half of that number. The quotient of a number by 3 is one-third of the number; by 4 is one-fourth of the number; by 5 is one-fifth; by 6 is one-sixth of the number, etc.

What is one-fourth of 12? What is one-sixth of 18? One-fifth of 20? One-seventh of 14? One-ninth of 27? One-third of 21? One-eighth of 40?

What is division? What is the dividend? What the divisor? What the quotient? Make the three signs of division, and illustrate each? What is the quotient of 1 divided by 2? Of 2 divided by 3? What is a fraction? How do we express the quotient of a less number divided by a greater? If 1 be divided into 2 equal parts, what is each part called? If into 3, what is each part called? What, if into 6 equal parts?

Examples for Oral Work.

76. *1.* How many apples, at one cent each, can you buy for 5 cents?

Solution.—As many apples as there are 1's in 5, or 5 apples.

2. How many marbles, at 2 cents each, can you buy for 4 cents?

Solution.—As many marbles as there are 2's in 4, or 2 marbles.

3. How many pears, at 3 cents each, can you buy for 6 cents ? How many for 18 *cts.* ?

4. How many peaches, at 4 cents each, can be bought for 12 cents? How many for 24 *cts.*?

5. If I divide 15 apples among 5 boys, giving each an equal number, how many apples will each boy receive?

6. If a man travel 6 miles an hour, how many hours will it take him to travel 18 miles? $18 \div 6 = ?$

7. In an apple orchard there are 21 trees, and 7 trees in each row; how many rows in the orchard? $21 \div 7 = ?$

8. A man paid $24 for 8 boxes of oranges; how much did he pay for each box? $24 \div 8 = ?$

9. How many pairs of boots, at $9 a pair, can be bought for $36? How many for $45? $\frac{36}{9} = ?$ $\frac{45}{9} = ?$

10. If I divide a line 1 foot long into 2 equal parts, how long is each part?

11. If I divide 1 apple into 3 equal pieces, what part of the apple is each piece ?

12. What is the quotient of 2 divided by 3? Of 3 divided by 4? Of 5 divided by 6? Of 7 divided by 8?

SHORT DIVISION AND LONG DIVISION.

77. There are two methods of performing the operations of division: ***1.*** *Short Division,* in which the divisor does not exceed 12; and ***2.*** *Long Division,* in which the divisor exceeds 12.

In **Short Division** much of the work is carried on mentally; in **Long Division,** the different steps of the operation are written out.

SHORT DIVISION.

78. Let it be required to divide 19,224 by 4.

ILLUSTRATION.
Dividend,
Divisor, 4) 19224
Quotient, 4806

EXPLANATION.—Because 1 is less than 4, we divide 19 by 4; this gives a quotient 4, and a remainder 3; we set 4 under the 9, and to 3 we annex the following figure of the dividend, which gives 32. The quotient of 32 divided by 4 is 8; this we set under the 2. Since 2, the next figure of the dividend, is less than 4, we put a cipher in the quotient, and to 2 we annex the following figure of the dividend, which gives 24. Dividing 24 by 4, we find a quotient 6, which we write under 4. Hence, the required quotient is 4806.

EXAMPLES IN SHORT DIVISION.

(*1.*)	(*2.*)	(*3.*)	(*4.*)
5) 785	6) 618	7) 1561	8) 2736

79. *When there is a Fraction in the quotient.*

Let it be required to divide 459 by 4.

ILLUSTRATION.
4) 459
114$\frac{3}{4}$

EXPLANATION.—Since 4 is contained in 4 once, we write 1 under 4 for the left-hand figure of the quotient. We multiply the divisor 4 by 1, and subtract the product mentally from 4 in the dividend, and have no remainder. We then divide 5 by 4 and obtain 1 for a quotient, which we multiply by the divisor and subtract the product mentally from 5; this leaves 1 for a remainder. To this remainder we annex 9, the next figure of the dividend, making 19. We divide 19 by 4, obtaining 4 for a quotient, which multiplied by the divisor, gives 16; this we subtract mentally from 19 and obtain 3 for a remainder. The whole dividend has now been divided by 4, except 3. We have learned that a less number can be divided by a greater by writing the divisor under the dividend with a line between; hence 3 divided by 4 is $\frac{3}{4}$, which we place at the right of 4 in the quotient, giving 114$\frac{3}{4}$; read one hundred fourteen and three fourths.

In this manner work the following

EXAMPLES.

(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)
7) 856	5) 324	3) 4762	9) 992	6) 3214
$122\frac{2}{7}$				

RULE FOR SHORT DIVISION.

I. Write the divisor on the left of the dividend, and separate them by a line.

II. Divide the first figure of the dividend by the divisor, and write the quotient below; or if the first figure is less than the divisor, divide the first two figures, and write the quotient under the second.

III. If there is a remainder after any division, annex to it the next figure of the dividend, and divide as before.

IV. If any partial dividend is less than the divisor, write 0 *for the quotient figure, and annex the next figure of the dividend, for a new dividend.*

V. If there is a remainder, after dividing the last figure, write the divisor under it, and annex the result to the quotient.

PROOF.—*Multiply the quotient by the divisor, and if the result is equal to the dividend, the work is correct.*

EXAMPLES.

10. Divide 5,408 by 2.
11. Divide 9,147 by 3.
12. Divide 16,146 by 5.
13. Divide 5,124 by 4.
14. Divide 63,241 by 5.
15. Divide 1,981 by 7.
16. Divide 3,475 by 9.
17. Divide 113,214 by 11.

LONG DIVISION.

80. Let it be required to divide 2,756 by 26.

ILLUSTRATION.

$$\begin{array}{l}
26\ )\ 2756\ (\ 106\\
\underline{26}\\
156\\
\underline{156}
\end{array}$$

EXPLANATION.—We first say, 26 in 27, once, and place 1 in the quotient. Multiplying the divisor by one, subtracting, and bringing down the 5, we have 15 for the first partial dividend. We then say, 26 in 15, 0 times, and place the 0 in the quotient. We then bring down the 6, and find that the divisor is contained in 156, 6 times.

If the dividend contains dollars and cents, point off two figures on the right of the quotient for cents. If it contains dollars, cents, and mills, point off three figures on the right of the quotient.

RULE FOR LONG DIVISION.

I. Find how many times the divisor is contained in the fewest possible figures on the left of the dividend, for the first figure of the quotient; multiply the divisor by this figure, and subtract the product from the figures used.

II. To the remainder annex the following figure of the dividend, and divide the result by the divisor, for the second figure of the quotient; or, if the result is less than the divisor, put a cipher in the quotient, annex another figure, and proceed as before.

III. Continue the operation till all the figures of the quotient have been found.

IV. If there is a remainder after the last figure is brought down, write the divisor under it and annex the result to the quotient.

The method of proof is the same as for short division.

In applying the preceding rule, it is convenient to write the divisor on the left and the quotient on the right of the dividend. Should there be a remainder after the last figure of the quotient is found, it is to be treated as explained in short division.

EXAMPLES.

Divide

1. 854 by 25.
2. 11,232 by 36.
3. 836 by 22.
4. 1,674 by 31.
5. 2,944 by 46.
6. 5,184 by 27.
7. 19,032 by 61.
8. 22,274 by 55.

Divide

9. 17,808 by 48.
10. 8,856 by 82.
11. 20,962 by 94.
12. 16,340 by 76.
13. $870 by 15.
14. $9,504 by $16.
15. $12,972 by 23.
16. 6,475 *lbs.* by 25.

Divide

17. 3,894 *lbs.* by 33.
18. 3,476 *yds.* by 44.
19. 35,638 by 103.
20. 29,890 by 122.
21. 13,610 by 214.
22. 2,636 by 47.
23. 3,009 by 32.
24. 3,060 by 235.

NOTE.—If the entire dividend is divided there is no *final remainder*, hence we do not speak of a remainder in connection with the answer. Thus, if 13 is divided by 2 the quotient is 6½, not "6 and a remainder 1."

In the latter case only 12 is divided by 2, the remainder 1 is not divided.

81. *When there are ciphers on the right of the divisor.*

Divide 354,216 by 100.

FIRST SOLUTION,

$$\begin{array}{l}
100)354216(3542\tfrac{16}{100} \\
\qquad \underline{300} \\
\qquad\; 542 \\
\qquad\; \underline{500} \\
\qquad\quad 421 \\
\qquad\quad \underline{400} \\
\qquad\qquad 216 \\
\qquad\qquad \underline{200} \\
\qquad\qquad\; 16
\end{array}$$

EXPLANATION.—By this solution it will be seen that if we had removed as many figures from the right of the dividend as there are ciphers on the right of the divisor, the remaining figures of the dividend would be the same as the integral figures of the quotient, and the figures removed are the last remainder. This remainder is divided by writing the divisor under it as directed in the second method of expressing division.

If the divisor alone terminates in ciphers, we point them off, and also point off the same number of figures from the right of the dividend; we then divide the remaining part of the dividend by the significant part of the divisor, and annex to the last remainder the figures pointed off from the dividend. This remainder, with the entire divisor written beneath, is annexed to the quotient, and becomes a part of it.

SECOND SOLUTION.

25,00)378,43(15$\frac{343}{2500}$
25
128
125
Rem. 343

EXPLANATION.—We point off the two ciphers in the divisor, and also two figures from the right of dividend; we then divide 378 by 25, which gives a quotient 15, and a remainder 3; to this we annex the figures cut off from the dividend, which gives 343; but, 343 has not been divided; to divide it, we write the entire divisor under it. Hence, the final result is 15$\frac{343}{2500}$.

EXAMPLES.

1. Divide 8,734 by 400.
2. Divide 34,121 by 6,000.
3. Divide 184,381 by 900.
4. Divide 37,564 by 2,500.
5. Divide 272,543 by 16,000.
6. Divide 36,452 by 1,500.

If both dividend and divisor terminate in ciphers, we strike out from each, as many as are common to both. Thus, the quotient of 16,000 by 400 is the same as the quotient of 160 by 4. Striking out two ciphers is the same as dividing both dividend and divisor by 100.

When dollars, cents, and mills are divided by 10, 100, 1000, etc., the point is moved to the left as many places as there are ciphers in the divisor. If there is not a sufficient number of figures at the left of the point, supply the deficiency by prefixing ciphers.

7. Divide 1,500 by 300.
8. Divide 21,000 by 700.
9. Divide 815,000 by 5,000.
10. Divide 62,500 by 250.
11. Divide 5,120 by 1,600.
12. Divide 67,470 by 3,000.

TEST QUESTIONS.

The quotient of any number by 2, is what part of that number? The quotient of any number by 3, is what part of that number? By 4? By 6? By 7? By 8? By 15? By 20? In methods of work there are how many cases in division? What are they? How are the operations carried on in short division? How in long division? Illustrate short division by an example of your own. Give the rule for short division. If the dividend is composed of dollars and cents, how many figures do you point off on the right of the quotient? If composed of dollars, cents, and mills, how many do you point off? Write an example in long division, perform the work, and explain each step in the process. Give the rule for long division. How do you prove division? How do you perform the work when there are ciphers on the right of the divisor? How, when there are ciphers on the right of both divisor and dividend?

PRACTICAL PROBLEMS.

82. In solving problems in division, we proceed as though both dividend and divisor were abstract numbers, and then determine the unit of the answer from the nature of the problem.

FOR ORAL WORK.

1. At $6 a yard, how many yards can be bought for $54?

SOLUTION.—As many as 6 is contained in 54, which is 9; hence, 9 yards can be bought for $54.

2. How many dozen eggs can be bought for 96 cents, at 12 cents a dozen? How many for 108 *cts.*?

3. If a man can dig a ditch 56 yards long in 7 days, digging an equal number each day, how many yards does he dig in a day?

4. At $6 a ton for coal, how many tons can be bought for $72? How many for $36? For $48?

5. If 7 yards of ribbon cost 70 cents, what will 1 yard cost? What will 2 yards cost? 5 yards?

FOR WRITTEN WORK.

6. How long will it take a man to walk 1,404 miles at the rate of 27 miles a day? How long, at the rate of 81 miles a day? At the rate of 108 miles a day?

7. A man earns $1,924 in 52 weeks; how much does he earn a week? How much if he earns $3,848 in 52 weeks? How much if he earns $5,772 per week?

8. How long will it take a steamer to sail 2,880 miles, if she sails 240 miles a day? How long, if she sails 120 miles a day? How long if she sails 60 miles a day?

9. There are 60 seconds in one minute; how many minutes in 8,640 seconds? How many in 4,320 seconds?

10. If 75 horses cost $21,225, what is the cost of each horse? What, if 150 horses cost $21,225?

FOR ORAL WORK.

83. *1.* If 9 *lbs.* of sugar cost $1.08, what will 1 *lb.* cost?

2. Paid $36 for 12 sheep; what did each sheep cost?

3. There are 4 gills in 1 pint; how many pints in 48 gills? In 96 gills? In 192 gills?

4. There are 8 quarts in 1 peck, how many pecks in 72 quarts? How many in 96 quarts? In 192 quarts?

5. If you have 7 pecks of nuts, and put them in three-quart bags, filling each bag, and then you give me what are over; how many bags will you need, and how many quarts will you give me.

FOR WRITTEN WORK.

6. If two persons start from the same point and travel in opposite directions, one at the rate of 17 miles a day, and the other at the rate of 25 miles a day, how long will it be before they are 504 miles apart?

7. If 3 horses cost $720; what will one horse cost, and what will 13 horses cost, at the same rate?

8. There are 24 hours in 1 day and 7 days in 1 week; how many weeks are there in 39,984 hours?

9. A farmer sold 4 pairs of oxen for $214 a pair, 13 cows for $43 each, and 2 horses for $118 each; after paying a debt of $251, he bought with the remainder 20 acres of land; what did the land cost per acre?

10. A clerk received $7,500 salary for 5 years services; he spent $3,900 for board, $1,210 for clothing, and deposited the rest in the bank; how much did he spend each year for board? how much each year for clothing? how much did he deposit each year in the bank?

FOR ORAL WORK.

84. *1.* If 3 quarts of berries cost 36 cts., what do 5 quarts cost?

SOLUTION.—If 3 quarts of berries cost 36 cts., 1 quart costs $\frac{1}{3}$ of 36 cts., or 12 cts., and 5 quarts cost 5 times 12 cts., or 60 cts.

2. If 4 yards of cloth cost $12, what do 9 yards cost?

3. Bought 9 barrels of flour at $6 a barrel, and paid for it in coal at $3 a ton; how many tons of coal did it take? (9 × 6) ÷ 3 = ?

4. In one gallon there are 4 quarts. If I buy a quart of molasses at 48 cts. a gallon, and pay a 25-cent piece, how much change should I receive? 25 — (48 ÷ 4).

5. How much will one-half a gallon of vinegar cost at 24 *cts.* a gallon? At 32 *cts.* a gallon?

6. How much does $\frac{1}{4}$ of an acre of land cost at $36 an acre? At $96 an acre? At $192?

7. 10 meters make one dekameter; how many dekameters in 84 meters? In 49 meters?

8. If you had $67, how much flour could you buy at $5 a barrel? How much at $7 a barrel?

9. Four pecks make 1 bushel. If you buy a bushel of apples for 84 cts., what is the cost of half a peck?

10. 5 men bought a horse for $75, paying equal shares; if they sell the horse for $40, how much will each man lose? ($75 — $40) ÷ 5 = ?

FOR WRITTEN WORK.

11. A grocer sold 64 *lbs.* of sugar at 14 cents a *lb.*, and 4 *lbs.* of tea at 96 cents a *lb.*, for which he was paid in butter at 32 cents a *lb.*; how many pounds of butter did he receive?

12. What number multiplied by 3 will give the same product as 27 multiplied by 7?

13. A grocer buys 7,381 *lbs.* of cheese, at 8 cents a pound, and pays for it in coffee at 22 cents a pound; how much coffee does he give for the cheese?

14. If flour costs $14 a barrel, how much can be bought for $1,358?

15. A merchant sold 4 pieces of cloth: The first two pieces contained 45 *yds.* each, the third contained 47 *yds.*, and the fourth contained 53 *yds.*; for the whole he received $760; how much did he receive per yard?

16. There are $750 in 4 bags; the first contains $115, the second contains $236; the third contains $60 less than the first and second together; how much does the fourth contain?

17. A grocer packs 789 *lbs.* of butter, which fills 17 tubs and 7 *lbs.* over; how much does he put in each tub?

18. A farmer bought a farm, for which he paid $18,050; he sold 50 acres for 60 dollars an acre, and the remainder at 50 dollars an acre; how much land did he buy?

19. A merchant bought a hogshead of molasses, containing 96 gallons, at 35 cents per gallon; but 26 gallons leaked out, and he sold the remainder at 50 cents per gallon; did he gain or lose, and how much?

20. Mr. Bailey has 7 calves, worth 4 dollars apiece, 9 sheep, worth 3 dollars apiece, and a fine horse, worth 375 dollars. He exchanges them for a yoke of oxen, worth 125 dollars, and a colt, worth 65 dollars, and takes the balance in hogs, at 8 dollars apiece; how many hogs does it take?

21. The distance from Chicago to San Francisco is 2,448 miles; how long will it take a man to walk the whole distance at the rate of 24 miles a day?

22. A man bought a farm for $3,612; he sold half of it at $56 an acre, and received $2,408 for the half he sold; how many acres did he buy, and what did he give per acre?

RECAPITULATION AND GENERAL PRINCIPLES.

NOTATION.

85. A **Unit** is one, or a single thing.

A **Number** is a unit, or a collection of units.

The **Simple Value** of a figure is the value it expresses when standing alone, or in the units place.

The **Local Value** is that which it has when standing in any particular place. Thus, the value taken of 2 in the first place is 2 *units*, in the second place it is 2 *tens*, in the third place it is 2 *hundreds*, and so on.

Every place in a number not occupied by a significant figure must be filled by a *cipher*.

A **Rule** is a brief direction for performing work.

A **Scale** is an order of progression on which any system of notation is founded.

A **Uniform Scale** is one in which the law of progression is the same throughout, as in the Arabic notation.

A **Varying Scale** is one in which the law of progression is changed at every step, as in the notation of English money.

ADDITION.

86. Only similar numbers can be added together.

SUBTRACTION.

87. The **Minuend** and **Subtrahend** must have the same unit, or they must be capable of being reduced to the same unit.

The same number added to or subtracted from both minuend and subtrahend, does not change the value of the remainder.

GENERAL PRINCIPLES.

MULTIPLICATION.

88. The **Multiplier** is always as an abstract number.

The **Multiplicand** and **Products** are like numbers.

The multiplier and multiplicand are together called **Factors** of the product.

Multiplying either factor by any number, *multiplies the product* by the same number.

The product of a number multiplied by itself is called the **Square of the number.**

Multiplying both factors by the same number is equivalent to multiplying the product by the square of that number.

Multiplication may be proved by dividing the product by either factor; if the quotient is equal to the other factor, the work is supposed to be right.

DIVISION.

89. Multiplying the dividend by a number, *multiplies the quotient* by that number.

Multiplying the divisor by a number, *divides the quotient* by that number.

Multiplying both dividend and divisor by the same number does not change the quotient.

Dividing the dividend by a number, *divides the quotient* by that number.

Dividing the divisor by a number, *multiplies the quotient* by that number.

Dividing both divisor and dividend by a number, does not change the value of the quotient.

When the quotient of one number divided by another is integral, the dividend is said to be *exactly* divisible by the divisor, and the divisor is called an **Exact Divisor** of the dividend.

REVIEW QUESTIONS ON THE FUNDAMENTAL RULES, PRINCIPLES, Etc.

Define arithmetic. Write a *unit.* Write a *number* greater than 1. Give an example of an abstract number. Give an example of a concrete number. Write a number containing six orders of units. Separate 123468975321 into periods, and name each period. What numbers can be added together? Give the rule for addition. What is an arithmetical scale? Give an example of a uniform scale. Name the given numbers in subtraction. Define minuend. Define remainder. Construct an example in subtraction. Work it and prove it, explaining each step in the process. From 9472, subtract 2645, and explain each step. What is multiplication? Work the following problem, explain the operation, and tell how many, and what fundamental rules are used in the solution: Two persons start from the same place, and travel in the same direction; one travels at the rate of 6 miles an hour, the other at the rate of 9 miles an hour: if they travel 8 hours a day, how far will they be apart at the end of 17 days? How far, if they travel in opposite directions? Prove the work. Define product; define factors; define multiplicand; define multiplier; define divisor; define dividend; define quotient. Make all the signs of division. Give the rule for long division. Wherein does short division differ from long division? If the divisor and dividend are concrete numbers, will the quotient be concrete or abstract? Divide 1,041,835 by 204, and explain each step in the operation. If the dividend is multiplied by any number, the divisor remaining unchanged, how is the quotient effected? If the divisor is multiplied, and the dividend remains unchanged, how is the quotient effected?

PROPERTIES OF NUMBERS.

DEFINITIONS.

90. Properties of a number are qualities that necessarily belong to it.

EXERCISES FOR ORAL WORK.

1. If 24 is the sum of two numbers, and one of them is 13, what is the other?

2. What two factors besides the number itself and 1, will produce the product 14? What will produce 15? 21?

3. What three numbers used as factors will produce the product 30? What three will produce 45? What 36?

DEFINITION.

91. An **Exact Divisor** of a number is one that will give an integral number only for a quotient; 2 is an exact divisor of 4, of 6, of 8, of 10, etc.

4. What is the smallest number except 1, that will exactly divide all of the numbers, 10, 12, 14 and 16?

5. What is the smallest number except 1, that will exactly divide 15, 12, 18, and 21?

6. What is the largest number that will exactly divide the numbers 12, 18, and 24?

7. What is the largest number that will exactly divide 18, 27, and 36?

8. Name an exact divisor of 6 and 12; of 9 and 18; of 24 and 36; of 24 and 32; of 28 and 36.

9. Name the largest exact divisor of 6 and 12; of 9 and 18; of 20 and 30; 24 and 32; 28 and 36.

DEFINITION.

92. A **Prime Number** is one that has no exact divisor except itself and one; as 2, 3, 5, 7.

1. Name all the prime numbers between 1 and 10; between 10 and 20; 20 and 30; 30 and 40; 40 and 50; 50 and 60; 60 and 70; 70 and 80; 80 and 90; 90 and 100.

2. Write all the prime numbers between 1 and 100.

DEFINITIONS.

An **Even Number** is a number exactly divisible by 2.

93. An **Odd Number** is one that is not exactly divisible by 2.

What is a composite number? Name the composite numbers between 1 and 20; name the prime factors of each.

The process of separating composite numbers into factors is called **Factoring.**

EXERCISES FOR ORAL WORK.

1. Find the prime factors of 4; of 6; of 8; of 12; of 15; of 20.

2. Find the prime factors of the *sum* of 4 and 6; 5 and 7; 8 and 7; 9 and 3; 10 and 5.

3. Find the prime factors of the product of 4 and 6; 3 and 9; 2 and 8; 4 and 5; 6 and 7.

4. What are the prime factors of 12? 10? 16? 20? 26? 28? 30? 32? 35? 36?

5. What are the prime factors of 21? 28? 32? 36? 38? 42? 45? 49? 50?

6. What are the prime factors of 27? 25? 35? 49? 44? 52? 60? 56?

Any composite number may be resolved into prime factors by the following

RULE.

Divide the number by one of its prime factors; then divide the quotient by one of its prime factors; and so on till a quotient is found that is prime; the several divisors and the last quotient are the required factors.

EXERCISES FOR WRITTEN WORK.

Let it be required to factor 130.

ILLUSTRATION.

$$\begin{array}{r} 2\underline{)130} \\ 5\underline{)65} \\ 13 \end{array}$$

$130 = 2 \times 5 \times 13$

EXPLANATION.—Dividing 130 by 2, we have 65 for a quotient; dividing this quotient by 5, we have 13 for a quotient, which is a prime number; hence, the required factors are 2, 5, and 13.

EXAMPLES.

Resolve the following numbers into prime factors:

1. 210.
2. 330.
3. 1,015.
4. 156.
5. 310.
6. 495.
7. 425.
8. 990.
9. 975.
10. 765.
11. 570.
12. 504.
13. 1,485.
14. 2,625.
15. 918.
16. 342.
17. 824.
18. 632.
19. 548.
20. 350.

EQUAL FACTORS.

The prime factors of 4 are 2 and 2; $2 \times 2 = 4$.

The prime factors of 8 are 2, 2 and 2; $2 \times 2 \times 2 = 8$.

The prime factors of 9 are 3 and 3; $3 \times 3 = 9$.

The prime factors of 16 are 2, 2, 2 and 2.

The number of times the same factor is used in producing a composite number is sometimes indicated by a figure written at the right and a little above the factor; thus, $3^2 = 9$; $2^3 = 8$; $4^2 = 16$; $2^4 = 16$.

In the last illustration the figure 2, standing above and to the right of 3, is called an **exponent**, and the product of the equal factors is called a **power.**

DEFINITIONS.

94. An **Exponent** is a number written at the right and a little above the number, to indicate how many times the number is used as a factor.

95. A **Power** is the product of any number of equal factors. Hence 4 is the second power of 2: and the expression $2^2 = 4$ is read *2 second power*, or *2 square, equals 4*. $2^3 = 8$ is read *2 third power*, or *2 cube*, equals 8.

Read the following

EXAMPLES.

(1.)	*(2.)*	*(3.)*	*(4.)*	*(5.)*	*(6.)*
$2^2 = 4$,	$2^3 = 8$,	$2^4 = 16$,	$3^2 = 9$,	$3^3 = 27$,	$3^4 = 81$.

What are properties of numbers? What is an exact divisor? What is a prime number? An even number? An odd number? What is factoring? Give the rule for resolving or separating a number into prime factors. What is an exponent? What is a power of a number?

CANCELLATION.

We have learned that multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient.

We can frequently take advantage of this principle to shorten our work.

Divide $29 \times 6 \times 8 \times 3 \times 10$ by $3 \times 6 \times 8$.

ILLUSTRATION.

$$\overset{1}{\cancel{3}} \times \overset{1}{\cancel{6}} \times \overset{1}{\cancel{8}}\ )\ 29 \times \overset{1}{\cancel{6}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{3}} \times 10 = 290.$$

EXPLANATION.—We first divide both divisor and dividend by 3, indicating the division by crossing 3 in each, and writing 1, the quotient of 3 divided by 3, over the factor crossed. In the same manner we divide by 6, and by 8. Our divisor becomes $1 \times 1 \times 1 = 1$, and our dividend becomes $29 \times 1 \times 1 \times 1 \times 10 = 29 \times 10 = 290$.

SECOND ILLUSTRATION.

$$\begin{array}{rcl} \cancel{645} & \div & \cancel{105} \\ \cancel{129} & \div & \cancel{21} \\ 43 & \div & 7 = 6\frac{1}{7}. \end{array}$$

EXPLANATION.—We divide both divisor and dividend by the common factor 5, and write the quotient beneath. This gives us 21 for a divisor and 129 for a dividend. Again we divide both by the common factor 3, and obtain 7 for a divisor and 43 for a dividend. 43 divided by 7 gives $6\frac{1}{7}$ for the quotient.

96. Cancellation in division is the process of shortening the operation by dividing both divisor and dividend by their common factor, or factors.

RULE FOR CANCELLATION.

Divide both divisor and dividend by all the common factors, crossing the numbers thus divided, writing the quotients over or under the numbers crossed; the product of these quotients and the remaining factors of the dividend, divided by the product of the quotients and remaining factors of the divisor, will give the required quotient.

EXAMPLES.

1. Divide $36 \times 7 \times 14$ by 2×3.

2. Divide $42 \times 8 \times 5 \times 12$ by $7 \times 5 \times 2$.

3. Divide $48 \times 14 \times 3$ by 8×7.

4. Divide $56 \times 18 \times 7 \times 3 \times 5$ by $7 \times 5 \times 2 \times 3$.

5. 390÷78.
6. 11850÷2370.
7. 2910÷2490.
8. 2625÷1485.
9. 5214÷4029.
10. 2190÷657.
11. 23485÷1830.
12. 3468÷124.
13. 2816÷12.

What is cancellation? What is the rule for cancellation? What is the object of cancellation?

GREATEST COMMON DIVISOR.

97. A **common divisor** of two or more numbers is the number that *will exactly divide them separately.*

98. The **greatest common divisor** of two or more numbers is the *greatest number* that will *exactly* divide them separately. Thus, 12 is the greatest common divisor of 24, 36, and 48.

There are two methods of finding the greatest common divisor: *1. By factors;* and *2. By continued division.*

METHOD BY FACTORS,

99. When the numbers can be resolved into factors, we may find their greatest common divisor by the following

RULE.

Resolve the numbers into prime factors, and find the product of those that are common to them all.

Let it be required to find the greatest common divisor of 240 and 330.

ILLUSTRATION. 240 resolved into its prime factors $= 2 \times 2 \times 2 \times 2 \times 3 \times 5$.

330 resolved into its prime factors $= 2 \times 3 \times 5 \times 11$.

2, 3 and 5 are common prime factors, hence the *greatest common divisor* $= 2 \times 3 \times 5 = 30$.

Find the greatest common divisor of

1. 6, 12, 30.
2. 28, 42, 70.
3. 84, 126, 210.
4. 15, 25, 30, 45.
5. 2010, 165, 525.
6. 3195, 1206.

METHOD BY CONTINUED DIVISION.

100. The greatest common divisor of two numbers can be found, without factoring, by the following

RULE.

I. Divide the greater number by the less; then take the divisor for a new dividend, and the remainder for a new divisor, and proceed as before.

II. Continue this operation till a remainder is found that will exactly divide the preceding divisor; this will be the required greatest common divisor.

ILLUSTRATION.

```
112)144(1
    112
     32)112(3
         96
         16)32(2
```

EXPLANATION.—We divide 144 by 112, and find a remainder 32; we next divide 112 by 32, and find a remainder 16, which exactly divides the preceding divisor; hence, 16 is the greatest common divisor of 112 and 144.

EXAMPLES.

Find the greatest common divisor of the following groups of numbers:

1. 216 and 316.
2. 39 and 192.
3. 1155 and 352.
4. 376 and 1645.
5. 1134 and 2079.
6. 3471 and 1869.

What is a common divisor of two or more numbers? The *greatest common divisor?* How many methods of finding the greatest common divisor? What are they? Give the rule for the method by factors. For the method by continued division.

LEAST COMMON MULTIPLE.

101. A **Multiple** of a number is a number that is exactly divisible by it. Thus, 18 is a multiple of 6.

A **Common Multiple** of two or more numbers is a number that is exactly divisible by each. Thus, 18 is a common multiple of 2, 3, and 6.

The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each. Thus, 12 is the least common multiple of 2, 3, and 6.

OPERATION OF FINDING THE LEAST COMMON MULTIPLE.

102. The least common multiple of two or more numbers may be found by the following

RULE.

I. Write the numbers in a line and then divide by any prime factor that is contained in two or more, writing the quotients and also the undivided numbers in the line below.

II. Then operate on this line in the same manner, and so continue till a line is found in which no two numbers have a common factor.

III. Find the continued product of the numbers in the last line and of the divisors used, and it will be the required least common multiple.

ILLUSTRATION.

$$\begin{array}{r|l} 3 & 3,\ 5,\ 6,\ 8 \\ \hline 2 & 1,\ 5,\ 2,\ 8 \\ \hline & 1,\ 5,\ 1,\ 4 \end{array}$$

$$3 \times 2 \times 5 \times 4 = 120$$

EXPLANATION. — Having written the numbers 3, 5, 6 and 8 in a line, we divide 3 and 6 by 3, placing the quotients underneath, and bringing down the undivided numbers; we then divide 2 and 8 by 2, bringing down as before; we then find the continued product of the numbers in the last line and of the divisors used; this gives 120, which is the required least common multiple.

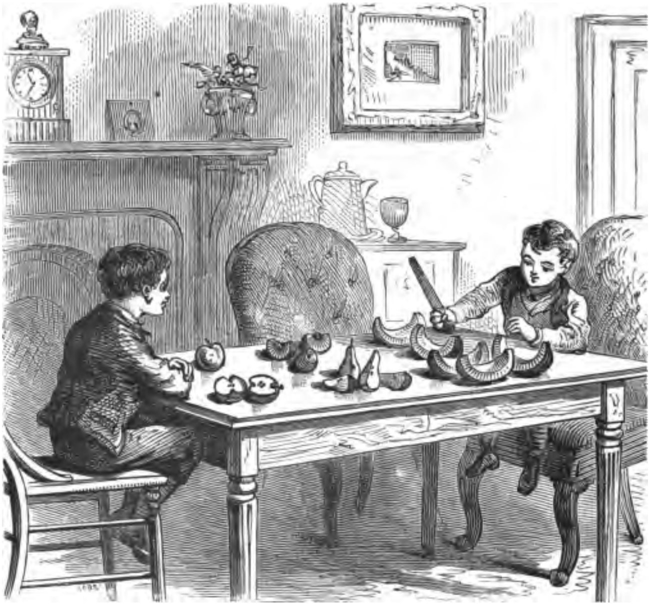
EXAMPLES.

Find the least common multiple of the following groups of numbers:

1. 5, 10, 15, and 20.
2. 10, 15, 24, and 30.
3. 8, 12, 18, and 24.
4. 6, 9, 12, and 15.
5. 15, 36, and 60.
6. 12, 14, 20, and 24.
7. 8, 9, 16, 24, and 27.
8. 7, 8, 15, 21, and 24.

REVIEW QUESTIONS.

What is a factor? What is a composite number? Illustrate. What is a prime number? Illustrate. What is factoring? What is the rule for finding the prime factors of a number? What is cancellation? What is it used for? How may division be simplified by cancellation? What is the greatest common divisor of two numbers? How many methods of finding it? Give the method by factors. By continued division. What is a multiple of a number? What is a common multiple of two or more numbers? The least common multiple of two or more numbers? Give the rule for finding the least common multiple. If the sum of two numbers and one of them is given, how will you find the other? If the difference between two numbers, and the less number be given, how will you find the greater? If the difference between two numbers and the greater be given, how will you find the less?



FORMATION OF FRACTIONS.

103. *1.* How many undivided apples are there in the picture?

What number will represent it?

Write the number by means of a figure.

This is an integer. An integer is a whole number.

2. One of the apples in the picture is divided into two equal parts. What part of the apple is one of the equal parts?

Write in figures the fractional number which will represent one of the halves. Write the fractional number that will represent two halves.

How many halves make one?

3. The peach is divided into three equal parts; what part of the peach is one of the equal parts?

Write in figures *one-third;* write *two-thirds.*

How many thirds make one?

4. The pear is divided into four equal pieces. What part of the pear is one of the pieces? What fraction will express two pieces? What, three pieces? What, four pieces? Write all these *fourths.*

5. The melon is divided into five equal pieces. What part of the melon is each piece?

Write in figures *one-fifth, two-fifths, three-fifths, four-fifths, five-fifths.*

Five-fifths are equal to what integral number?

A **Fraction** is one or more equal parts of a unit.

104. One of the equal parts into which an *integral unit* is divided is called a **Fractional Unit.**

In writing fractions we use one of the methods of indicating division, but we call the divisor and dividend by different names. The dividend, or number above the line, we call the *numerator;* the divisor, or number below the line, we call the *denominator.*

105. The **Denominator** shows into how many equal parts the integral unit is divided.

106. The **Numerator** shows how many of these equal parts are expressed by the fraction.

107. The **Value of a Fraction** is the quotient obtained by dividing the numerator by the denominator.

108. The **Terms of a Fraction** are the numerator and the denominator.

What is an integer? A fraction? A fractional unit? An integral unit? What are the terms of a fraction? What is the value of a fraction? What does the denominator show? The numerator? Which corresponds to the dividend? Which to the divisor?

Read the following fractions, and tell which is the numerator and which the denominator, and what each numerator and each denominator shows.

$\frac{7}{8}$, $\frac{13}{20}$, $\frac{12}{15}$, $\frac{5}{99}$, $\frac{9}{17}$, $\frac{7}{50}$, $\frac{7}{63}$, $\frac{3}{100}$, $\frac{7}{25}$, $\frac{8}{21}$, $\frac{4}{70}$.

EXAMPLES IN WRITING FRACTIONS.

1. Write 5 of the 6 equal parts of 1.

2. Write 12 of the 17 equal parts of 1.

3. If the fractional unit is one-twentieth, express 6 fractional units; express, also, 12 and 18 fractional units.

4. If the fractional unit is one-36th, express 32 fractional units; also, 6, 8, 12, 15, 21.

5. If the fractional unit is one-fortieth, express 9 fractional units; also, 16, 25, 69, 75, 36, 40, 18.

6. Write *forty-nine* / *hundredths.*

7. Write *three hundred and sixty-one* / *forty-sevenths.*

8. Write *seven thousand six hundred and fifteen* / *nine hundred and fifteenths.*

9. Write *six thousand four hundred* / *elevenths.*

10. Write *six thousand two hundred and forty-two* / *three hundred and fifty-thirds.*

NOTE.—In the preceding examples, and in all similar examples in this book, the sign / separates the nmmerator from the denominator.

KINDS OF FRACTIONS.

109. A **Proper Fraction** is one in which the numerator is less than the denominator; as, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$.

110. An **Improper Fraction** is one in which the numerator is equal to or greater than the denominator; as, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{8}{2}$, $\frac{9}{9}$.

If the numerator is equal to the denominator, the value of the fraction is equal to 1; thus, $\frac{4}{4} = 1$.

A proper fraction is less than 1; an improper fraction is equal to or greater than 1.

111. A **Mixed Number** is a number composed of a whole number, and a fraction; as, $2\frac{1}{2}$, $5\frac{3}{8}$.

112. A **Simple Fraction** is one in which both terms are whole numbers; as, $\frac{3}{7}$, $\frac{14}{5}$.

113. A **Complex Fraction** is one in which one term, at least, is either a fraction, or a mixed number; as, thus, $\frac{2\frac{1}{2}}{7}$, $\frac{3\frac{1}{4}}{4\frac{1}{2}}$, $\frac{\frac{2}{3}}{5}$ are complex fractions.

114. A **Compound Fraction** is a fraction of a fraction, or several fractions connected by the word of; as, $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{2}{3}$ of $2\frac{1}{2}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$.

115. The **Reciprocal of any number** is 1 divided by that number. The reciprocal of 4 is $1 \div 4$, or $\frac{1}{4}$.

116. The **Reciprocal of any fraction** is 1 divided by that fraction. It is equivalent to the fraction inverted. The reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3}$, or $\frac{3}{2}$.

117. A Fraction is inverted by causing its terms to change places; thus, $\frac{3}{4}$ inverted is $\frac{4}{3}$.

118. The **Analysis of a fraction** consists in naming its *integral unit,* the *kind* of fraction, its *terms,* its *fractional unit,* the *number of fractional units* and its *value.*

ILLUSTRATION.—In the fraction $\frac{3}{5}$, 1 is its integral unit; it is a simple, proper fraction, 5 is its denominator, and 3 is its numerator, $\frac{1}{5}$ is its fractional unit, 3 is the number of fractional units, and $\frac{3}{5}$ of 1 is its value.

Write the reciprocal of 2; of $\frac{2}{3}$; of $\frac{5}{8}$; of $\frac{3}{4}$; of $\frac{9}{11}$.
Analyze the fractions $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{6}{11}$, $\frac{14}{15}$, and $\frac{17}{15}$.
Invert the fractions $\frac{5}{8}$ and $\frac{3}{7}$, and read them.

PRINCIPLES.

119. From the nature of a fraction, we have the following principles:

1. Multiplying the numerator of a fraction by any number is equivalent to multiplying the fraction by that number.

2. Dividing the numerator of a fraction by any number is equivalent to dividing the fraction by that number.

3. Multiplying the denominator of a fraction by any number is equivalent to dividing the fraction by that number.

4. Dividing the denominator of a fraction by any number is equivalent to multiplying the fraction by that number.

5. Multiplying both terms of a fraction by the same number does not change the value of the fraction.

6. Dividing both terms of a fraction by the same number does not change the value of the fraction.

What is a proper fraction? What is an improper fraction? What is the value of a fraction when the terms are equal? What is a mixed number? What is a simple fraction? What is a complex fraction? What is a compound fraction? What is the reciprocal of a number? What is the reciprocal of a fraction? What is the value of a fraction? What is the analysis of a fraction? State the principles in the order in which they are placed on page 108.

REDUCTION.

120. Reduction is the operation of changing the form of a number without altering its value.

The following cases of reduction of fractions depend on the principles given in Art. 119.

121. *To reduce a fraction to higher terms.*

1. One-half is equal to how many sixths?

Solution.—Since 1 is equal to $\frac{6}{6}$, $\frac{1}{2}$ is equal to $\frac{1}{2}$ of $\frac{6}{6}$, or $\frac{3}{6}$.

2. One-half of an apple is equal to how many fourths?

Solution.—Since 1 is equal to $\frac{4}{4}$, $\frac{1}{2}$ is equal to $\frac{1}{2}$ of $\frac{4}{4}$, or $\frac{2}{4}$.

3. One-fourth of a dollar is how many eighths?

Solution.—Since 1 is equal to $\frac{8}{8}$, $\frac{1}{4}$ is equal to $\frac{1}{4}$ of $\frac{8}{8}$, or $\frac{2}{8}$.

4. Express $\frac{3}{4}$ in terms twice as large.

Solution.—Multiply both terms by 2; thus, $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.

RULE.

A fraction is reduced to higher terms by multiplying both the numerator and denominator by the same number.

122. *To reduce a fraction to lower terms.*

We learned in Art. 119, Principle 6, that dividing both terms of a fraction by the same number does not change the value of the fraction.

1. Reduce $\frac{6}{8}$ to a fraction whose terms are $\frac{1}{2}$ as great.

SOLUTION. $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$.

2. How many thirds of an apple are equal to $\frac{8}{12}$ of an apple ?

SOLUTION.—Since $\frac{1}{3}$ is equal to $\frac{4}{12}$, there will be as many thirds as there are $\frac{4}{12}$ in $\frac{8}{12}$, or two-thirds.

RULE

For reducing a fraction to lower terms.

Divide both numerator and denominator by the same number.

EXAMPLES.

1. Reduce $\frac{15}{20}$ to an equivalent fraction whose terms shall be $\frac{1}{5}$ as great.

2. How many fourths of a peach are $\frac{6}{12}$ of a peach?

3. Reduce $\frac{25}{35}$ to *sevenths.* Reduce $\frac{15}{25}$ to *fifths.*

4. How many *fifths* are equal to $\frac{36}{45}$?

5. How many *eighths* are equal to $\frac{30}{24}$?

123. *To reduce a fraction to its lowest terms.*

RULE.

I. Resolve the terms into prime factors, and cancel all that are common to both; multiply the remaining factors of the numerator together for a new numerator, and the remaining factors of the denominators for a new denominator; or,

II. Divide both terms of the fraction by their greatest common divisor.

EXAMPLES.

1. Reduce $\frac{12}{18}$ to its lowest terms.

SOLUTION. $\frac{12}{18} = \frac{\cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 3} = \frac{2}{3}$. *Ans.*

2. Reduce $\frac{24}{60}$ to its lowest terms.

3. Reduce $\frac{35}{135}$ to its lowest terms.

4. Reduce the following fractions to their lowest terms:

5. $\frac{132}{180}$.
6. $\frac{138}{150}$.
7. $\frac{805}{875}$.
8. $\frac{39}{243}$.
9. $\frac{96}{144}$.
10. $\frac{330}{78}$.
11. $\frac{1008}{875}$.
12. $\frac{616}{504}$.
13. $\frac{330}{858}$.
14. $\frac{242}{21}$.
15. $\frac{840}{40}$.
16. $\frac{384}{32}$.

If the terms cannot be factored by inspection, work by the second rule.

17. $\frac{1872}{2016}$.
18. $\frac{3069}{3168}$.
19. $\frac{363}{605}$.
20. $\frac{2587}{3383}$.
21. $\frac{5661}{6327}$.
22. $\frac{1089}{1573}$.
23. $\frac{1309}{1785}$.
24. $\frac{350}{1015}$.
25. $\frac{805}{875}$.
26. $\frac{504}{1845}$.
27. $\frac{3168}{5148}$.
28. $\frac{1050}{1386}$.
29. $\frac{3508}{3876}$.
30. $\frac{4950}{7700}$.
31. $\frac{1628}{2244}$.

What is reduction? How is a fraction reduced to higher terms? How is a fraction reduced to lower terms? Give the rule for reducing fractions to their *lowest* terms.

124. *To reduce an improper fraction to a whole or mixed number.*

How many integral units are there in $\frac{12}{2}$?

ILLUSTRATION.—Since 1 is equal to $\frac{2}{2}$, $\frac{12}{2}$ are equal to as many ones as there are $\frac{2}{2}$ in $\frac{12}{2}$, equal to 6.

RULE.

Divide the numerator by the denominator.

EXAMPLES.

1. Reduce $\frac{7}{3}$ to a mixed number. *Ans.* $2\frac{1}{3}$.

2. Reduce $\frac{38}{7}$ to a mixed number. *Ans.* $5\frac{3}{7}$.

Reduce the following to mixed numbers:

3. $\frac{41}{9}$.
4. $\frac{311}{12}$.
5. $\frac{480}{11}$.
6. $\frac{326}{13}$.
7. $\frac{223}{15}$.
8. $\frac{442}{7}$.
9. $\frac{752}{13}$.
10. $\frac{75}{22}$.
11. $\frac{222}{28}$.
12. $\frac{302}{11}$.
13. $\frac{849}{8}$.
14. $\frac{72413}{720}$.

125. *To reduce an integer to a simple fraction having a given denominator.*

In any number there are twice as many halves as whole ones, three times as many thirds, four times as many fourths, etc.

1. How many halves in 2 apples?

ILLUSTRATION.—Since in 1 apple there are $\frac{2}{2}$, in 2 apples there are twice $\frac{2}{2} = \frac{4}{2}$.

2. In 5 bushels of wheat, how many thirds?

ILLUSTRATION.—Since in 1 bushel there are $\frac{3}{3}$, in 5 bushels there are five times $\frac{3}{3} = \frac{15}{3}$.

3. Reduce 8 to a fraction whose denominator is 5.

ILLUSTRATION.—In 1 there are $\frac{5}{5}$, in 8 there are 8 times $\frac{5}{5} = \frac{40}{5}$.

From these illustrations we deduce the following

RULE.

Multiply the integer by the given denominator and write the product over that denominator.

EXAMPLES.

4. Reduce 12 to *thirds.* Reduce 9 to *halves.*

5. Reduce 9 to *eighths.* Reduce 7 to *thirds.*

6. Reduce 12 to *sixteenths;* 13 to *fourths.*
7. Reduce 5 to *fifteenths;* 15 to *fifths.*
8. Reduce 14 to *eighteenths;* 17 to *ninths.*
9. Reduce 75 to *fifths;* 84 to *thirds.*
10. Reduce 115 to *fourths;* 112 to *sixths.*
11. Reduce 86 to *ninths;* 73 to *eighths.*

126. *To reduce a mixed number to an improper fraction.*

Let it be required to reduce $4\frac{7}{8}$ to *eighths.*

ILLUSTRATION.

$4 = \frac{32}{8}$, hence

$$4\tfrac{7}{8} = \frac{32}{8} + \frac{7}{8} = \frac{39}{8}$$

EXPLANATION.—$4=\frac{32}{8}$, hence, $4\frac{7}{8}=\frac{32}{8}$ and $\frac{7}{8}$; but 32 *eighths* and 7 *eighths* make 39 *eighths,* that is, $4\frac{7}{8}=\frac{39}{8}$. Here we have multiplied 4 by 8 and to the product we have added 7; we have then written the sum over 8. Hence the

RULE.

Multiply the integral part by the denominator of the fractional part, add the numerator to the product, and write the sum over the denominator.

EXAMPLES.

1. Reduce $7\frac{1}{8}$ to an improper fraction. *Ans.* $\frac{57}{8}$.
2. Reduce $8\frac{1}{4}$ to an improper fraction. *Ans.* $\frac{33}{4}$.
3. Reduce $11\frac{3}{7}$ to an improper fraction. *Ans.* $\frac{80}{7}$.

Reduce the following mixed numbers to improper fractions:

4. $102\frac{5}{9}$.
5. $236\frac{4}{10}$.
6. $215\frac{5}{14}$.
7. $49\frac{1}{8}$.
8. $63\frac{4}{11}$.
9. $88\frac{3}{14}$.
10. $97\frac{5}{16}$.
11. $84\frac{3}{20}$.
12. $114\frac{21}{44}$.

13. How many *twelfths* in $18\frac{2}{12}$? In $21\frac{5}{12}$? In $35\frac{7}{12}$?
14. How many *fifteenths* in $17\frac{3}{15}$? In $27\frac{4}{15}$? In $36\frac{5}{15}$?

127. *To reduce fractions to equivalent fractions having a common denominator.*

RULE.

Multiply both terms of each fraction by the product of the denominators of all the other fractions.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to equivalent fractions having a common denominator. Reduce $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to equivalent fractions having a common denominator. Reduce $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{4}{9}$.

3. Reduce $\frac{3}{8}$, $\frac{7}{11}$, $\frac{8}{13}$, and $\frac{5}{7}$ to equivalent fractions having a common denominator. Reduce $\frac{5}{8}$, $\frac{6}{11}$, $\frac{7}{10}$, and $\frac{4}{15}$.

4. Reduce $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ each, to twelfths.

5. Reduce $\frac{7}{9}$, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{11}$, $\frac{10}{13}$ to equivalent fractions having a common denominator.

The common denominator is any *multiple of all the denominators.* The *least common denominator* is the least multiple of the denominators.

1. Reduce $\frac{2}{3}$, $\frac{5}{7}$, and $\frac{3}{14}$ to equivalent fractions having the least common denominator.

ILLUSTRATION.

$$\begin{array}{l}
7\underline{)3,\quad 7,\quad 14}\qquad \frac{2}{3} = \frac{28}{42}\\
\;\;\;3,\quad 1,\quad\; 2\qquad\;\; \frac{5}{7} = \frac{30}{42}\\
7 \times 3 \times 2 = 42.\quad \frac{3}{14} = \frac{9}{42}
\end{array}$$

EXPLANATION.—Find the common multiple of the denominators, which is 42. One-third of 42 is 14. Now, if 3, the denominator of the first fraction, is multiplied by 14, we obtain 42; but if the denominator is multiplied by 14, the numerator must be multiplied by the same number to preserve the value; hence we have $\frac{2}{3} = \frac{28}{42}$. In the same manner $\frac{5}{7}$ and $\frac{3}{14}$ are reduced to *forty-seconds.*

128. ***Fractions may be reduced to equivalent fractions having the least common denominator by the following***

RULE.

Find the least common multiple of all the denominators for a common denominator; then multiply both terms of each fraction by the quotient of the least common multiple by the denominator of that fraction.

EXAMPLES.

Reduce the following groups to their least common denominator:

2. $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{7}{18}$. *Ans.* $\frac{54}{90}$, $\frac{40}{90}$, $\frac{35}{90}$.

3. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{10}$. *Ans.* $\frac{20}{30}$, $\frac{24}{30}$, $\frac{25}{30}$, and $\frac{21}{30}$.

4. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{2}{9}$. *Ans.* $\frac{60}{90}$, $\frac{75}{90}$, $\frac{63}{90}$, and $\frac{20}{90}$.

5. $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{16}$.

6. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

7. $\frac{7}{25}$, $\frac{11}{30}$, and $\frac{8}{15}$.

8. $\frac{2}{15}$, $\frac{5}{16}$, and $\frac{11}{20}$.

9. $\frac{7}{9}$, $\frac{11}{15}$, and $\frac{11}{12}$.

10. $\frac{3}{5}$, $\frac{2}{9}$, $\frac{2}{3}$, and $\frac{7}{15}$.

11. $\frac{2}{27}$, $\frac{3}{54}$, $\frac{7}{9}$, and $\frac{5}{18}$.

12. $\frac{5}{32}$, $\frac{3}{28}$, and $\frac{11}{20}$.

If fractions have a common denominator, they have the same fractional unit.

TEST QUESTIONS.

What is a fraction? What is a proper fraction? What is a simple fraction? What is an improper fraction? How is an improper fraction reduced to a whole or mixed number? What is a mixed number? How is a mixed number reduced to an improper fraction? Give the rule for reducing an integer to a simple fraction having a given denominator. How are fractions having different denominators reduced to equivalent fractions having a common denominator? Give the rule for reducing fractions to equivalent fractions having the least common denominator. What is a complex fraction?

ADDITION OF FRACTIONS.

129. Addition of Fractions is the operation of finding the sum of two or more fractions.

1. Henry gave $\frac{1}{2}$ of an apple to John, and $\frac{1}{2}$ of an apple to Charles: how many half apples did he give away?

SOLUTION. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$.

2. James bought $\frac{1}{4}$ of a bushel of wheat in the morning, $\frac{1}{4}$ in the afternoon, and $\frac{1}{4}$ in the evening: what part of a bushel did he buy during the day?

SOLUTION. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

3. How many fifths in $\frac{1}{5} + \frac{2}{5} + \frac{3}{5}$? *Ans.* $\frac{6}{5} = 1\frac{1}{5}$.

To add fractions they must have common denominators.

4. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

ILLUSTRATION.

$$\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{40}{60} + \frac{45}{60} + \frac{48}{60}.$$

$$\frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = 2\frac{13}{60}.$$

EXPLANATION.—Here we reduce the fractions to equivalent fractions having a common denominator, add their numerators and place their sum over the common denominator; then, because the sum is an improper fraction, we reduce it to a mixed number.

From the above explanation, we have the following

RULE.

I. When the fractions have the same denominator, add the numerators, and place their sum over the common denominator. If the result is an improper fraction, reduce it to a whole or mixed number.

II. When they have not the same denominator, reduce them to a common denominator, and add as before.

EXAMPLES FOR ORAL WORK.

1. Add $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{5}{4}$.

2. What is the sum of $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$, and $\frac{6}{5}$?

3. If Jane pays $\frac{1}{5}$ of a dollar for paper, $\frac{1}{10}$ of a dollar for a pencil, and $\frac{3}{10}$ for a bottle of ink, how much does she pay for the whole?

4. Henry spends $\frac{1}{6}$ of a dollar at one time, \$$1\frac{5}{6}$ at another, \$$2\frac{1}{6}$ at another; how much does he spend?

5. How many twenty-fourths are there in $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$? How many in $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$? How many in $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{2}{3}$.

6. Add $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{10}$.

7. Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{2}{9}$.

8. Add $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{2}{16}$.

9. Add 9, $\frac{2}{3}$, $\frac{1}{15}$, $\frac{5}{6}$, and $\frac{2}{3}$.

10. Add 12, $\frac{3}{10}$, $\frac{6}{20}$, and $\frac{7}{30}$.

11. Add $\frac{4}{9}$, $\frac{17}{90}$, $\frac{13}{30}$, and 15.

EXAMPLES FOR WRITTEN WORK.

Add the following groups of fractions:

1. $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{16}$.

2. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

3. $\frac{7}{25}$, $\frac{11}{30}$, and $\frac{8}{15}$.

4. $\frac{2}{15}$, $\frac{5}{16}$, and $\frac{11}{20}$.

5. $\frac{7}{9}$, $\frac{11}{15}$, and $\frac{11}{12}$.

6. $\frac{3}{5}$, $\frac{2}{9}$, $\frac{2}{3}$, and $\frac{7}{15}$.

7. $\frac{2}{27}$, $\frac{3}{54}$, $\frac{7}{9}$, and $\frac{5}{18}$.

8. $\frac{5}{32}$, $\frac{3}{28}$, and $\frac{11}{20}$.

9. $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.

10. $\frac{1}{15}$, $\frac{5}{21}$, and $\frac{7}{18}$.

To add mixed numbers, we add the integral parts and the fractional parts separately, and then find the sum of the results; or, reduce them to improper fractions and add by the last rule.

Add the following groups of fractions:

11. $2\frac{1}{2}$, $3\frac{3}{4}$, and $4\frac{7}{8}$.

12. $3\frac{3}{7}$, $4\frac{11}{14}$, and $5\frac{23}{28}$.

13. $\frac{7}{18}$, $2\frac{1}{4}$, 8, and $8\frac{5}{6}$.

14. $\frac{1}{3}$, $2\frac{1}{5}$, $7\frac{1}{9}$, and $4\frac{5}{12}$.

15. $4\frac{1}{9}$, $7\frac{1}{5}$, and $3\frac{11}{45}$.

16. $2\frac{5}{7}$, $\frac{3}{7}$, $4\frac{9}{14}$, and 8.

17. $3\frac{1}{5}$, $2\frac{3}{10}$, $5\frac{11}{15}$, and $6\frac{7}{20}$.

18. $13\frac{1}{9}$, $5\frac{3}{10}$, and $\frac{11}{45}$.

19. $3\frac{2}{3}$, $4\frac{2}{5}$, and $\frac{1}{15}$.

20. $2\frac{1}{8}$, $7\frac{4}{5}$, and $2\frac{3}{10}$.

PRACTICAL PROBLEMS.

1. A. bought 3 pieces of cloth; the first contained $39\frac{1}{2}$ *yds.*, the second $38\frac{1}{4}$ *yds.*, and the third $40\frac{7}{8}$ *yds.*; how many yards in all? *Ans.* $118\frac{5}{8}$.

2. John bought a sled for \$$3\frac{1}{2}$, a pair of skates for \$$1\frac{7}{8}$, and a cap for \$$4\frac{3}{16}$; what did they all cost him?

Ans. \$$9\frac{9}{16}$.

3. A farmer bought a farm, of which $27\frac{1}{4}$ *acres* were woodland, $49\frac{3}{8}$ *acres* pasture, $17\frac{4}{5}$ *acres* ploughland, and $36\frac{1}{10}$ *acres* meadow; what was the content of the farm?

Ans. $125\frac{21}{40}$ *acres.*

4. A man spent at a store \$$7\frac{3}{8}$ for a barrel of flour, \$$6\frac{1}{6}$ for sugar, \$$5\frac{4}{5}$ for tea, and had \$$2\frac{1}{2}$ remaining; how much had he at first?

5. A merchant sold cloth for \$$37\frac{1}{2}$, ribbons for \$$2\frac{7}{8}$, thread for \$$1\frac{1}{16}$, and pins for \$$\frac{7}{8}$; how much did they all amount to?

6. Find the sum of $7\frac{1}{2}$ *lbs.*, $4\frac{1}{4}$ *lbs.*, $3\frac{7}{8}$ *lbs.*, and $4\frac{3}{16}$ *lbs.*

7. Add $3\frac{1}{4}$ *yds.*, $7\frac{1}{8}$ *yds.*, $5\frac{3}{16}$ *yds.*, and $4\frac{1}{2}$ *yds.*

8. How many *yds.* in 4 pieces of cloth measuring respectively $27\frac{1}{2}$ *yds.*, $37\frac{3}{4}$ *yds.*, $39\frac{1}{5}$ *yds.*, and $30\frac{9}{10}$ *yds.*?

9. How many tons of coal in 5 loads weighing respectively $1\frac{1}{8}$, $1\frac{2}{5}$, $1\frac{3}{10}$, $\frac{1}{2}$, and $1\frac{1}{4}$ tons?

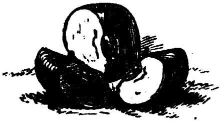
10. How many dollars will pay for a coat worth \$$14\frac{3}{4}$, a hat worth \$$5\frac{1}{2}$, a vest worth \$$6\frac{1}{4}$, a pair of pants worth \$8, and a pair of boots worth \$$9\frac{1}{4}$?

11. How many *pounds* of butter in 4 tubs, weighing respectively $27\frac{1}{3}$ *lbs.*, $34\frac{3}{8}$ *lbs.*, $32\frac{4}{5}$ *lbs.*, and $29\frac{2}{9}$ *lbs.*?

What is addition? What kinds of fractions can be added together? Give the rule for addition of fractions.

SUBTRACTION OF FRACTIONS.

130. Subtraction of Fractions is the operation of finding the difference between two fractions.



James has $\frac{2}{3}$ of an orange; if he gives $\frac{1}{3}$ to his sister, what part of the orange has he left?

ILLUSTRATION. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$.

Charles has $\frac{3}{4}$ of a dollar and gives a poor woman $\frac{1}{4}$ of a dollar; what part of a dollar has he left?

ILLUSTRATION. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

Henry bought $\frac{2}{3}$ of a pound of raisins and gave William $\frac{1}{2}$ of a pound; what part of a pound had he left?

Reduce $\frac{2}{3}$ and $\frac{1}{2}$ to sixths. $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$.

What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Reduce $\frac{1}{3}$ to sixths. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.

From the above illustrations we have the following

RULE.

I. When the fractions have the same denominator, subtract the less numerator from the greater, and place the difference over the common denominator.

II. When they have not the same denominator, reduce them to a common denominator, and subtract as before.

EXAMPLES FOR ORAL WORK.

1. From $\frac{3}{4}$ subtract $\frac{1}{2}$.
2. From $\frac{5}{6}$ take $\frac{2}{3}$.
3. From $\frac{7}{11}$ take $\frac{2}{7}$.
4. From $\frac{17}{20}$ take $\frac{4}{5}$.
5. From $\frac{3}{5}$ take $\frac{1}{3}$.
6. From $\frac{1}{2}$ take $\frac{1}{5}$.
7. From $\frac{2}{3}$ take $\frac{1}{7}$.
8. From $\frac{23}{24}$ take $\frac{43}{48}$.
9. From $\frac{17}{2}$ take $\frac{4}{9}$.
10. From $\frac{22}{5}$ take $\frac{7}{3}$.
11. From $\frac{19}{9}$ take $\frac{11}{10}$.
12. From $\frac{17}{30}$ take $\frac{2}{5}$.

When there are mixed numbers, change both to improper fractions, and subtract as before.

EXAMPLES FOR WRITTEN WORK.

13. From $\frac{2}{7}$ take $\frac{4}{15}$.
14. From $3\frac{7}{15}$ take $1\frac{2}{3}$.
15. From $5\frac{7}{25}$ take $2\frac{7}{35}$.
16. From $2\frac{5}{6}$ take $1\frac{7}{12}$.
17. From 7 take $\frac{2}{5}$.
18. From $3\frac{3}{7}$ take $1\frac{4}{5}$.

PRACTICAL PROBLEMS.

1. Find the difference between \$$\frac{7}{8}$ and \$$\frac{1}{4}$. *Ans.* \$$\frac{5}{8}$.

2. I had \$$2\frac{1}{4}$, and spent \$$1\frac{1}{8}$; how much had I left?

3. A farmer had $34\frac{1}{2}$ *tons* of hay, and sold $11\frac{3}{8}$ *tons*; how much had he left?

4. What is the difference between $8\frac{1}{3}$ *yds.* and $3\frac{5}{6}$ *yds.*?

5. The sum of two numbers is $12\frac{7}{16}$; one of the numbers is $7\frac{4}{7}$; what is the other?

6. James, Joseph, and Daniel owned three farms, whose total area was $475\frac{9}{10}$ acres. Daniel had 16 acars more than Joseph, and Joseph 24 acars more than James; how many acres had each in his farm?

7. A tailor had a piece of cloth containing $29\frac{1}{2}$ *yds.*; *he cut off* $4\frac{1}{2}$ *yds.* to make a coat and $1\frac{1}{2}$ *yds.* to make a *pair of pants.* How many *yds.* were left?

8. A housekeeper bought 6 mahogany chairs for \$$23\frac{7}{8}$, and gave for them, 2 ten-dollar and 1 five-dollar bill; what change ought she to receive?

What is subtraction? What fractions can be subtracted? Give the rule for subtraction of fractions. When there are mixed numbers, what is the method?

MULTIPLICATION OF FRACTIONS.

131. Multiplication of Fractions is the operation of finding the product of two factors when one or both are fractions.

132. *To multiply a fraction by a whole number.*

What part of an apple is twice $\frac{1}{4}$ of it?

SOLUTION. $\frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$. (Art. 119, Principle 1.)

What part of an orange is 3 times $\frac{1}{4}$ of it?

SOLUTION. $\frac{1}{4} \times 3 = \frac{3}{4}$.

If each of 4 boys gives me $\frac{1}{8}$ of a pear, what part of a pear shall I then have?

SOLUTION. $\frac{1}{8} \times 4 = \frac{4}{8} = \frac{1}{2}$.

If we had divided the denominator by 4, we should have obtained the same final result. Art. 119, Principle 4.

RULE.

Multiply the numerator by the whole number and write the product over the denominator, or divide the denominator by the whole number and write the quotient under the numerator.

EXAMPLES FOR ORAL WORK.

1. If one yard of cloth cost $\frac{5}{8}$ of a dollar, what will 4 yards cost?

2. Multiply $\frac{47}{49}$ by 7.

3. Multiply $\frac{175}{45}$ by 9.

4. Multiply $\frac{127}{15}$ by 5.

5. Multiply $\frac{369}{147}$ by 3.

EXAMPLES FOR WRITTEN WORK.

6. If 1 dollar will buy $\frac{5}{9}$ of a cord of wood, how much will 15 dollars buy? How much 18 dollars?

7. At $\frac{5}{6}$ of a dollar a pound, what will 12 pounds of tea cost? What will 24 *lbs.* cost?

8. If a horse eats $\frac{3}{4}$ of a bushel of oats in a day, how much will 18 horses eat in a day?

9. What will 64 pounds of cheese cost, at $\frac{3}{22}$ of a dollar a pound? What will 66 *lbs.* cost?

NOTE.—When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results; or, reduce the mixed number to an improper fraction, and multiply.

10. At $2\frac{3}{8}$ cents a pound, what will 8 pounds of chalk cost? What will 10 *lbs.* cost?

11. If a man earns $3\frac{9}{10}$ dollars a day, how much will he earn in 15 days? How much in 21 days?

12. If a family consumes $5\frac{4}{7}$ barrels of flour in 1 year, how much would they consume in 9 years? In 12 years?

133. *To multiply a whole number by a fraction.*

At $2 a yard, what will $\frac{1}{2}$ yard of cloth cost?

SOLUTION. $\frac{1}{2}$ of $2 = 2 \times \frac{1}{2} = \frac{2}{2} = 1$.

At $15 a ton, what will $\frac{4}{5}$ of a ton of hay cost?

SOLUTION.

$15 \times \frac{4}{5} = \frac{15}{5} \times 4 = 12.$

EXPLANATION.—1st. Four *fifths* of a ton will cost 4 times as much as 1 *fifth* of a ton; if 1 ton cost 15 dollars, 1 *fifth* will cost $\frac{1}{5}$ of 15 dollars, or 3 dollars, and $\frac{4}{5}$ will cost 4 times 3 dollars, which are 12 dollars. Hence the

RULE.

Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator;

Or, *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

NOTE.—Cancel, when possible.

EXAMPLES FOR MENTAL WORK.

1. What is the product of $9 \times \frac{1}{3}$? $8 \times \frac{1}{2}$? $12 \times \frac{1}{4}$?

2. What is the product of $20 \times \frac{1}{4}$? $28 \times \frac{1}{4}$? $25 \times \frac{1}{5}$?

3. What is the product of $63 \times \frac{3}{7}$? Of $45 \times \frac{4}{9}$?

4. What is the product of $28 \times \frac{3}{4}$? Of $25 \times \frac{2}{5}$?

EXAMPLES FOR WRITTEN WORK.

5. Multiply 33 by $\frac{5}{8}$.

6. Multiply 37 by $\frac{5}{6}$.

7. Multiply 191 by $\frac{2}{9}$.

8. Multiply 525 by $\frac{3}{5}$.

9. Multiply 65 by $\frac{6}{7}$.

10. Multiply 58 by $\frac{7}{8}$.

11. Multiply 466 by $\frac{6}{7}$.

12. Multiply 646 by $\frac{7}{8}$.

To multiply a whole number by a mixed number, multiply by the fraction and then by the integral part, and find the sum of the products.

13. Multiply 981 by $4\frac{1}{3}$.

ILLUSTRATION.

$$\begin{array}{r} 3)981 \\ 4\frac{1}{3} \\ \hline 327 \\ 3924 \\ \hline 4251 \end{array}$$

Explanation.—Multiplying 981 by $\frac{1}{3}$ is the same as dividing it by 3; this gives 327; multiplying 981 by 4, we find 3924, and the sum of the results is equal to 4251, which is the required product.

14. Multiply 67 by $9\frac{1}{2}$.

15. Multiply 9 by $12\frac{2}{3}$.

16. Multiply 108 by $12\frac{4}{9}$.

17. Multiply 1,464 by $3\frac{1}{4}$.

134. *To multiply one fraction by another.*

1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

ILLUSTRATION.

$$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}.$$

Explanation.—To multiply $\frac{3}{4}$ by $\frac{5}{7}$, we multiply $\frac{3}{4}$ by 5, and then divide the result by 7. To multiply $\frac{3}{4}$ by 5, we multiply its numerator by 5, (Art. 119, Prin. 1); to divide the result by 7, we multiply the denominator by 7, (Art. 119, Prin. 3), that is, we multiply 3 by 5 and 4 by 7.

From this illustration and explanation we deduce the following

RULE.

Reduce the fractions (when necessary) to equivalent simple ones; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

EXAMPLES FOR ORAL WORK.

2. Multiply $\frac{2}{5}$ by $\frac{4}{7}$. *Ans.* $\frac{8}{35}$.

3. Multiply $\frac{5}{3}$ by $\frac{2}{7}$.

4. Multiply $\frac{3}{8}$ by $\frac{5}{7}$.

5. Multiply $\frac{3}{2}$ by $\frac{5}{7}$.

6. Multiply $\frac{3}{5}$ by $\frac{7}{5}$.

7. Multiply $\frac{3}{5}$ by $\frac{3}{5}$.

8. Multiply $\frac{5}{2}$ by $\frac{7}{3}$.

9. Multiply $\frac{12}{7}$ by $\frac{3}{8}$.

10. *Multiply* $\frac{7}{36}$ by $\frac{24}{17}$.

11. Multiply $\frac{100}{27}$ by $\frac{15}{25}$.

12. Multiply $\frac{28}{5}$ by $\frac{35}{7}$.

EXAMPLES FOR WRITTEN WORK.

If there are any factors common to the numerator and the denominator of the indicated product, cancel them before performing the multiplication.

13. What is the product of $\frac{1}{2}\times\frac{7}{8}\times\frac{4}{5}\times\frac{9}{4}$?

14. Find the result of $\frac{9}{10}\times\frac{2}{3}\times1\frac{1}{4}$.

15. What is the product of $\frac{3}{8}\times\frac{6}{7}\times\frac{7}{9}\times\frac{9}{11}\times\frac{11}{12}\times\frac{15}{17}$?

16. Find the product of $\frac{5}{9}\times\frac{13}{15}\times\frac{3}{5}\times\frac{9}{20}\times\frac{6}{7}$.

RULE FOR REDUCING A COMPOUND FRACTION.

A compound fraction is reduced to a simple one by multiplying its numerators together for a numerator, and the denominators together for a denominator.

17. Reduce to a simple fraction $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$.

18. Reduce $\frac{1}{7}$ of 14 of $\frac{3}{5}$ of 15 of $\frac{6}{7}$ to a simple fraction.

Multiply

19. $\frac{1}{8}$ of 3, by $\frac{1}{6}$ of $15\frac{1}{3}$.

20. $\frac{2}{3}$ of $\frac{5}{2}$ of $\frac{3}{5}$ by $4\frac{1}{6}$.

21. $14\frac{5}{6}$ of 9, by $6\frac{3}{7}$.

22. $\frac{3}{7}$ of 6 of $\frac{5}{4}$, by $\frac{8}{5}$ of 4.

23. $\frac{1}{9}$ of $\frac{3}{5}$ of $\frac{7}{8}$ of 40.

24. $\frac{1}{2}$ of $5\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{1}{21}$.

25. $\frac{9}{11}$ of $\frac{44}{3}$ of $\frac{7}{20}$ of 18.

26. $\frac{6}{13}$ of $4\frac{1}{2}$ of $\frac{3}{23}$ of 9.

To multiply two mixed numbers together, reduce both to simple fractions, and then multiply by the rule for multiplication of fractions.

27. Multiply $7\frac{1}{2}$ by $7\frac{1}{2}$. *Ans.* $\frac{15}{2}\times\frac{15}{2}=56\frac{1}{4}$.

28. Multiply $2\frac{2}{3}$ by $4\frac{1}{2}$. *Ans.* $\frac{8}{3}\times\frac{9}{2}=12$.

29. Multiply $4\frac{1}{8}$ by $3\frac{7}{11}$.

30. *Multiply* $3\frac{2}{3}$ by $5\frac{1}{9}$.

31. Multiply $12\frac{1}{2}$ by $87\frac{1}{5}$.

32. Multiply 643 by $4\frac{2}{5}$.

What is multiplication of fractions? Give the rule for multiplying a fraction by a whole number. When the multiplicand is a mixed number, how do you proceed? Give the rule for multiplying a whole number by a fraction. A whole number by a mixed number. One fraction by another. If there are factors common to both terms, what should be done? How is a compound fraction reduced to a simple one? How multiply two mixed numbers together?

PRACTICAL PROBLEMS.

1. What will $2\frac{1}{2}$ bushels of corn cost at \$$1\frac{1}{2}$ per bushel?

2. A man owned $\frac{2}{3}$ of a farm and sold $\frac{5}{8}$ of his interest; what part of the farm did he sell? *Ans.* $\frac{5}{12}$.

3. What is the cost of $37\frac{1}{2}$ *lbs.* of lard at $17\frac{1}{2}$ *cts.* a *lb.*?

4. Find the value of 876 bushels of oats at $62\frac{1}{2}$ *cts.* a *bu.*

5. What is the cost of $16\frac{3}{4}$ *yds.* of ribbon at $62\frac{1}{2}$ *cts.* a *yd.*?

6. What is the cost of $19\frac{1}{2}$ *yds.* of muslin at $22\frac{1}{2}$ *cts.* a *yd.*?

7. A farmer sold $151\frac{1}{4}$ bushels of wheat at \$$1\frac{1}{2}$ per bushel; how much did he receive for it?

8. A grocer sold $36\frac{1}{4}$ *lbs.* of tea at $87\frac{1}{2}$ *cts.* a pound; how much did it bring?

9. If a man can earn \$$2\frac{1}{2}$ per day, how much can he earn in $31\frac{1}{4}$ days?

10. What is the product of $6\frac{1}{5}$, $2\frac{7}{9}$, and $\frac{1}{4}$ of 12?

11. What will 24 yards of cloth cost, at \$$3\frac{3}{4}$ a yard?

12. What will $6\frac{2}{3}$ bushels of wheat cost, at \$$3\frac{3}{4}$ a bushel?

13. A horse eats $\frac{3}{14}$ of $\frac{7}{9}$ of 12 tons of hay in three months; how many tons does he consume?

14. If $\frac{2}{3}$ of $\frac{5}{8}$ of a dollar buy a bushel of corn, what will $\frac{7}{10}$ of $\frac{6}{11}$ of a bushel cost?

15. What cost $5\frac{2}{3}$ gallons of molasses, at $96\frac{1}{2}$ *cts.* a gallon?

16. What will $7\frac{11}{12}$ dozen candles cost, at \$$\frac{3}{16}$ per dozen?

DIVISION OF FRACTIONS.

135. Division of Fractions is the operation of finding the quotient when either the divisor or dividend, or both, are fractions.

136. *To divide a fraction by a whole number.*

1. If 2 bushels of potatoes cost \$$\frac{4}{5}$, what will 1 bushel cost?

1 bushel will cost $\frac{1}{2}$ as much as 2 bushels, $=\frac{2}{5}$ of a dollar.

2. If 3 bushels of apples cost $\frac{6}{7}$ of a dollar, what will 1 bushel cost? $\frac{6}{7} \div 3 = \frac{2}{7}$. (Art. 119, Prin. 2.)

3. If 4 quarts of beans cost $\frac{2}{3}$ of a dollar, what will 1 quart cost? $\frac{2}{3} \div 4 = \frac{2}{12} = \frac{1}{6}$. (Art. 119, Prin. 3.)

RULE,

Divide the numerator by the whole number, and write the quotient over the denominator; or, multiply the denominator by the whole number, and write the product under the numerator.

EXAMPLES FOR ORAL WORK.

1. Divide $\frac{12}{11}$ by 6.

2. Divide $\frac{18}{37}$ by 9.

3. Divide $\frac{18}{19}$ by 6.

4. Divide $\frac{24}{91}$ by 12.

5. If 6 horses eat $\frac{9}{10}$ of a ton of hay in 1 month, how much does each horse eat?

6. If 9 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost? What will 3 *yds.* cost?

7. If 1 yard of cloth cost 4 dollars, how much can be bought for $\frac{8}{9}$ of a dollar? How much for $\frac{11}{9}$ of a dollar?

8. If 5 pounds of coffee cost $\frac{15}{16}$ of a dollar, what will 1 pound cost? What will 2 *lbs.* cost?

9. At $6 a barrel, what part of a barrel of flour can be bought for $\frac{3}{5}$ of a dollar? For $$\frac{4}{5}$? For $11?

10. If 10 bushels of barley cost $3\frac{1}{3}$ dollars, what will 1 bushel cost? What will 4 bushels cost?

EXAMPLES FOR WRITTEN WORK.

1. Divide $\frac{405}{19}$ by 15.

2. Divide $\frac{450}{531}$ by 75.

3. Divide $\frac{15}{33}$ by 20.

4. Divide $\frac{126}{651}$ by 27.

5. Divide $\frac{561}{784}$ by 25.

6. Divide $\frac{172}{395}$ by 13.

7. Divide $3\frac{1}{8}$ by 15.

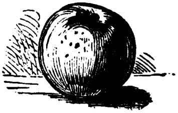
8. Divide $4\frac{1}{2}$ by 16.

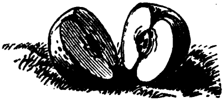
9. If 21 pounds of raisins cost $4\frac{2}{3}$ dollars, what will 1 pound cost? What will $4\frac{1}{2}$ *lbs.* cost? $7\frac{3}{4}$ *lbs.*?

10. If 12 men consume $6\frac{2}{5}$ pounds of meat in a day, how much does 1 man consume?

NOTE.—Reduce the mixed numbers to improper fractions, and divide as in the case of a simple fraction.

137. *To divide a whole number by a fraction.*





1. How many halves in 1 apple?

2. How many fourths in 1 apple?

3. How many fifths in 1 apple?

4. How many $\frac{1}{2}$ bushels of wheat can we empty into a box which will hold 2 bushels?

Since we can empty 1 bushel 2 times, we can empty $\frac{1}{2}$ bushel twice 2 times, or 4 times. $2 \div \frac{1}{2} = 2 \times \frac{2}{1} = 4.$

RULE.

I. Invert the terms of the divisor, and multiply the whole number by the resulting fraction; or,

II. Divide the dividend by the numerator of the divisor, and multiply the quotient by the denominator.

EXAMPLES FOR ORAL WORK.

1. Divide 6 by $\frac{2}{3}$. $6 \div \frac{2}{3} = \frac{6}{2} \times 3 = 9.$

2. Divide 12 by $\frac{6}{7}$.

3. Divide 18 by $\frac{9}{11}$.

4. Divide 25 by $\frac{5}{7}$.

5. Divide 36 by $\frac{12}{13}$.

6. At $\frac{11}{12}$ of a dollar a yard, how many yards of cloth can be bought for 9 dollars?

7. If a man travel $\frac{7}{8}$ of a mile in one hour, how long will it take him to travel 10 miles?

8. If $\frac{5}{8}$ of a ton of hay is worth 9 dollars, what is a ton worth?

EXAMPLES FOR WRITTEN WORK.

1. Divide 17 by $\frac{11}{5}$.

2. Divide 100 by $\frac{10}{13}$.

3. Divide 500 by $\frac{3}{8}$.

4. Divide 27 by $\frac{9}{10}$.

5. Divide 750 by $\frac{100}{147}$.

6. Divide 100 by $6\frac{7}{10}$.

7. Divide 75 by $10\frac{2}{3}$.

8. Divide 97 by $8\frac{1}{3}$.

138. *To divide one fraction by another.*

ILLUSTRATION.

$$\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}.$$

EXPLANATION.—Here we have to divide $\frac{3}{7}$ by $\frac{4}{5}$; we multiply $\frac{3}{7}$ by 5 and then divide the result by 4; but this is the same thing as multiplying $\frac{3}{7}$ by $\frac{5}{4}$. Hence, we invert the divisor, that is, we cause its terms *to change places*, and then multiply the dividend by the result.

RULE.

Reduce mixed numbers and complex fractions to simple ones. Invert the terms of the divisor and proceed as in multiplication.

EXAMPLES FOR ORAL WORK.

1. Divide $\frac{2}{3}$ by $\frac{5}{7}$.

2. Divide $\frac{3}{8}$ by $\frac{2}{11}$.

3. Divide $\frac{2}{9}$ by $\frac{3}{5}$.

4. Divide $\frac{4}{7}$ by $\frac{3}{5}$.

5. How much cheese can be bought for $\frac{7}{8}$ of a dollar, at $\frac{2}{11}$ of a dollar a pound?

EXAMPLES FOR WRITTEN WORK.

1. Divide $\frac{3}{5}$ by $\frac{8}{77}$.

2. Divide $\frac{6}{35}$ by $\frac{4}{9}$.

3. Divide $\frac{1}{2}$ of $\frac{3}{5}$ by $\frac{8}{15}$.

4. Divide $2\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{2}{7}$.

5. Divide $\frac{1}{2}$ of $\frac{3}{5}$ by $\frac{7}{15}$.

6. Divide $2\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{2}{7}$.

7. Divide $\frac{15}{8} \times \frac{36}{5}$ by $\frac{22}{9}$.

8. Divide $7\frac{1}{2}$ by $10\frac{2}{3}$.

9. Divide $9\frac{7}{8}$ by $8\frac{1}{3}$.

10. Divide $\frac{2}{3}$ of 16 by $\frac{1}{2}$ of $\frac{1}{7}$.

COMPLEX FRACTIONS.

139. Complex fractions are reduced to their simplest forms by the operations of Reduction and Division.

1. Reduce $\dfrac{2\frac{2}{5}}{4\frac{1}{2}}$ to its simplest form.

ILLUSTRATION.

$$\frac{2\frac{2}{5}}{4\frac{1}{2}} \times \frac{10}{10} = \frac{24}{45} = \frac{8}{15}.$$

EXPLANATION. — Multiplying both terms of the complex fraction by 10, the least common multiple of the denominators of the fractions $\frac{2}{5}$ and $\frac{1}{2}$, we have $\frac{24}{45}$, (Art. 119, Prin. 5.) This reduced to its lowest terms is $\frac{8}{15}$. Hence, for the reduction of a complex fraction to its simplest form, we have the following

RULE.

I. Reduce the numerator and denominator each (when necessary) to equivalent simple fractions; then, multiply both terms of the complex fraction by the least common multiple of the denominators: or,

II. Divide the numerator of the complex fraction by the denominator.

EXAMPLES.

1. Reduce $\frac{6\frac{2}{3}}{\frac{2}{3}}$ to a simple fraction. $\frac{6\frac{2}{3} \times 3}{\frac{2}{3} \times 3} = \frac{20}{2} = 10.$

2. Reduce $\frac{7\frac{11}{12}}{3\frac{5}{6}}$ to a simple fraction. $\frac{7\frac{11}{12} \times 12}{3\frac{5}{4} \times 12} = \frac{95}{51}.$

3. Reduce $\frac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}}$ to a simple fraction.

Note.—In the third example we reduce the terms of the complex fraction to simple fractions.

Solution. $\frac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}} = \frac{\frac{6}{45} \times 180}{\frac{22}{36} \times 180} = \frac{24}{140} = \frac{12}{55}.$

Reduce the following complex fractions to simple ones.

4. $\frac{2\frac{1}{4}}{3\frac{1}{5}}$.

5. $\frac{3\frac{1}{2}}{4}$.

6. $\frac{7}{4\frac{1}{4}}$.

7. $\frac{8\frac{1}{12}}{3}$.

8. $\frac{14\frac{9}{10}}{\frac{7}{9} \text{ of } 15}$.

9. $\frac{214\frac{3}{4}}{25\frac{11}{12}}$.

10. $\frac{\frac{4}{7} \text{ of } \frac{2}{3} \text{ of } 5\frac{1}{4}}{1\frac{5}{16} \text{ of } 48}$.

11. $\frac{\frac{5}{6} \text{ of } \frac{7}{8} \text{ of } 4\frac{1}{3}}{\frac{8}{9} \text{ of } 27}$.

TEST QUESTIONS.

What is division of fractions? Give the rule for dividing a fraction by a whole number. How do you divide a whole number by a fraction? What is understood by *inverting* the terms of a fraction? Give the rule for dividing one fraction by another. What is a complex fraction? By what operations are complex fractions reduced to simple ones? Give the rule. What is a problem?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

1. How many pairs of gloves can I buy for \$2, at $\frac{1}{3}$ of a dollar a pair? How many for \$3? For \$6?

2. If a man walks 1 mile in $\frac{2}{9}$ of an hour, how many miles will he walk in 1 hour? How many in 2 hours?

3. If a man can hoe $\frac{1}{3}$ of a field of corn in one day, in how many days can he hoe $\frac{3}{4}$ of a field? In how many days can he hoe the whole field?

4. If 5 *yds.* of muslin cost \$$1\frac{1}{8}$, what will 1 *yd.* cost? What will 2 *yds.* cost? What will 10 *yds.* cost?

5. If 5 pounds of coffee cost $\frac{10}{13}$ of a dollar, what will 1 pound cost? What will 2 *lbs.* cost? What 5 *lbs.*?

FOR WRITTEN WORK.

1. How many barrels of apples can be bought for \$$20\frac{1}{2}$, if one barrel costs \$$2\frac{3}{4}$? *Ans.* $7\frac{5}{11}$ *barrels.*

2. How many bags of flour will \$$70\frac{3}{8}$ buy, if each bag costs \$$2\frac{1}{4}$? How many if each bag costs \$$4\frac{1}{2}$?

3. If \$$37\frac{3}{8}$ is divided equally among 6 persons, how much will each receive? $37\frac{3}{8} \div 12 = ?$

4. A merchant sold from a piece of cloth $4\frac{1}{2}$ *yds.* to one customer, $5\frac{7}{8}$ to a second, and $3\frac{1}{4}$ to a third; how much did he sell in all?

5. A grocer sold $10\frac{1}{2}$ *lbs.* of sugar at $12\frac{1}{2}$ cents per *lb.*; and $14\frac{1}{4}$ *lbs.* at $16\frac{1}{8}$ cents; what did he receive for it all?

6. A grocer had a barrel of sugar that weighed $348\frac{1}{4}$ *lbs.*, from which he sold $58\frac{1}{4}$ *lbs.* to one customer, $27\frac{1}{2}$ *lbs.* to another, and $64\frac{3}{4}$ *lbs.* to a third; how much *had he then* remaining?

7. What is the cost of $7\frac{1}{2}$ tons of coal, at \$$7\frac{1}{2}$ per ton?

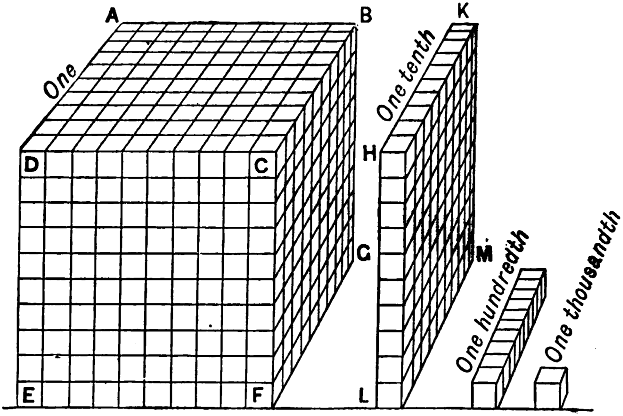
8. If 11 men consume a barrel of flour in $14\frac{1}{2}$ days, how long will $5\frac{1}{4}$ barrels last them?

9. If one man earns \$$2\frac{1}{2}$ in 1 day, and another earns \$$3\frac{1}{4}$ in 1 day, how much will both earn in $3\frac{1}{2}$ days?

10. A dealer buys corn at the rate of \$$\frac{7}{8}$ per bushel, and sells it at \$$1\frac{1}{20}$ per bushel; how much does he make on $142\frac{1}{2}$ bushels?

REVIEW QUESTIONS.

What is a fractional unit? Illustrate. What is a fraction? The denominator? The numerator? Terms? How are common fractions written? How read? Write a fraction in which the integral unit is divided into 7 parts, and 3 of those parts are expressed. Tell which is its numerator and which its denominator. What is a proper fraction? An improper fraction? A mixed number? A simple fraction? A complex fraction? A compound fraction? Write an improper fraction. A mixed number. A proper fraction. Are the proper and improper fractions that you have written simple fractions? Write a complex fraction. Reduce it to a simple fraction. Write a compound fraction. Reduce it to a simple fraction. Is it a proper or an improper fraction? State the six principles used in fractions. What is reduction? Give the rule for reducing a whole number to a simple fraction having a given denominator. A mixed number to a simple fraction. An improper fraction to a mixed number. A simple fraction to its lowest terms. A compound fraction to a simple one. Fractions to their least common denominator. A complex fraction to a simple one. Define addition of fractions. Give the rule for addition of fractions. Define subtraction of fractions. Give the rule for subtraction of fractions. If a boy spends $\frac{1}{3}$ of his money for a book, and $\frac{2}{5}$ of it for a slate, what part of his money has he left? Define multiplication of fractions. Give the rule for multiplying a fraction by a whole number. For multiplying a whole number by a fraction. One fraction by another. Define division of fractions. Give the rule for dividing a fraction by a whole number. A whole number by a fraction. A fraction by a fraction.



FORMATION OF DECIMAL FRACTIONS.

140. If a block is divided into 10 equal parts, each part will be $\frac{1}{10}$ of the block.

If one of the equal parts is divided into 10 equal parts, each of the parts will be $\frac{1}{100}$ of the original block; $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

If one of the last parts is again divided into 10 equal parts, each part will be $\frac{1}{1000}$ of the original block; $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{1000}$.

EXAMPLES.

1. If an apple is divided into 10 equal parts, what is each part called? $\frac{1}{10}$ of $1 = ?$

2. If $\frac{1}{10}$ of an apple is divided into 10 equal parts, what is each part called? $\frac{1}{10}$ of $\frac{1}{10} = ?$

Write each of the parts you have mentioned.

3. What part of one is $\frac{1}{10}$ of $\frac{1}{100}$? Write it.
4. What part of one is $\frac{1}{10}$ of $\frac{1}{1000}$? Write it.

These fractions are called decimal fractions.

DEFINITIONS.

141. A **decimal fraction** is a fraction whose denominator is 1, with one or more ciphers annexed.

One of the decimal divisions of an integral unit is a **decimal unit,** as $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

142. *Manner of writing Decimals.*

Integers increase from the right to the left, and decrease from the left to the right by a *scale of tens.*

In the integral number 1111, 1 hundred is $\frac{1}{10}$ of 1 thousand, 1 ten is $\frac{1}{10}$ of 1 hundred, and 1 unit is $\frac{1}{10}$ of 1 ten.

We will place a point (.) after the unit figure, and continue the scale; the next expression is $\frac{1}{10}$, which we indicate by 1 on the right of the point, thus 1111.1, and read it one-tenth; the next is $\frac{1}{100}$, which we indicate by 1 in the second place on the right of the point, thus 1111.11, and read *it* one-hundredth; $\frac{1}{1000}$ is expressed by 1 in the third place on the right of the point, and so on.

We have two methods of writing decimal fractions.

1. Both numerator and denominator may be written.

2. The numerator alone may be written, and the denominator may be indicated by a point, (.), called the **decimal point.**

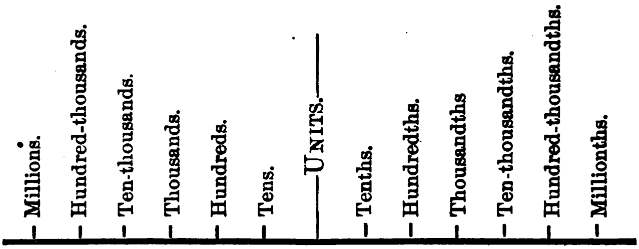
NOTATION OF DECIMALS.

143. To write a decimal fraction by the second method, we write the numerator; and if the number of figures is equal to the number of ciphers in the denominator, prefix the decimal point, Thus $\frac{3}{10} = .3$, $\frac{32}{100} = .32$. If the number of figures is less than the number of ciphers in the denominator, we make them equal by prefixing ciphers, and then prefix the decimal point.

Thus, $\frac{3}{100} = .03$, $\frac{52}{1000} = .052$.

Ciphers that immediately follow the decimal point are called **prefixed ciphers.** Ciphers that follow the last significant figure are called **terminal ciphers.**

An improper decimal fraction, written by means of the decimal point, is expressed as a mixed number; $\frac{23}{10}$ is written thus, 2.3, and read two and three-tenths. Such expressions are called **Mixed Decimals.**



DECIMAL SCALE

RULE FOR NOTATION OF DECIMALS.

Write the number of tenths in the first decimal place, the number of hundredths in the second place, the number of thousandths in the third place, and so on.

EXAMPLES

Write the following fractions, omitting the denominators.

(*1.*) $\frac{4}{10}$. (*2.*) $\frac{3}{100}$. (*3.*) $\frac{12}{1000}$. (*4.*) $\frac{113}{10000}$. (*5.*) $\frac{2}{10000}$. (*6.*) $\frac{372}{10}$ (*7.*) $\frac{1345}{100}$

144. *Numeration of Decimals.*

ILLUSTRATION.

Tenths.	Hundredths.	Thousandths.	10-thousandths.	100-thousandths.	Millionths.
.0	0	0	3	7	5

To read .000375, we commence at the decimal point and numerate, as shown in the illustration. The lowest order of units indicates the denominator; we read it 375 *millionths.*

Every fraction has a numerator and a denominator expressed or implied.

When a decimal fraction is written by means of the decimal point, its denominator is indicated by the lowest decimal unit; hence, to read decimals requires two numerations, one from the left to the right to determine the denominator, and one from the right to the left to read the numerator.

RULE FOR READING DECIMALS.

Read the numerator as if it were an integer, naming its denominator from the decimal unit of the right-hand figure.

EXAMPLES.

Read the following decimals.

1. .003.
2. .07.
3. .000123.
4. .005.
5. .032.
6. .314.
7. .50103.
8. 20256.
9. .135.
10. .001,325.
11. 7.814,3.
12. 6.010406.

Write the following numbers in figures, then numerate them, and read them.

13. Forty-one, and *three / tenths.*

14. Sixteen, and *three / millionths.*

15. Five, and *nine / hundredths.*

16. Sixty-five, and *fifteen / thousandths.*

17. Eighty, and *three / millionths.*

18. Two, and *three / hundred millionths.*

19. Four hundred, and *ninety-two / thousandths.*

20. Thirty-nine, and *six hundred and forty / thousandths.*

21. Three thousand, and *eight hundred and forty / millionths.*

22. *Six hundred and fifty / thousandths.*

Note.—In the preceding examples, decimals are in *italics.* The sign / separates the numerator from the denominator.

PRINCIPLES.

145. From the nature of decimals we also have the following principles:

1. *Annexing ciphers to a decimal does not change its value.*

2. *Removing ciphers from the right of a decimal does not change its value.*

3. *Annexing decimal ciphers to an integer does not change its value.*

What is a decimal fraction? One hundred is what part of one thousand? Ten is what part of one hundred? One is what part of ten? One-*tenth* is what part of one? One-hundredth is what part of one-tenth? One-thousandth is what part of one-hundredth? Write a decimal scale that will show the order of units from one-millionth to one million inclusive. How many methods have we of writing decimal fractions? Illustrate and explain both methods. Give the rule for notation of decimals. Numerate the decimal .234758. Read it. Give the rule for reading decimals. State the three principles given in Art. 145.

ADDITION OF DECIMALS.

146. Addition of decimals is the operation of finding the sum of two or more decimals.

1. What is the sum of $\frac{1}{10}$ and $\frac{3}{10}$? $\frac{1}{10} + \frac{3}{10} = \frac{4}{10}$. This expressed by means of the decimal point, is written thus, .4, and read four-tenths.

2. Add .1, .3, and .5. $.1 + .3 + .5 = .9$.

3. **What is the sum of .21 and .35?** $.21 + .35 = .56$.

4. If an arithmetic cost 5 tenths of a dollar, and a grammar 8 tenths, what do they both cost?

$$\frac{5}{10} + \frac{8}{10} = \frac{13}{10} = 1.3.$$

5. What is the sum of 37.04, 704, 3 and .0376?

ILLUSTRATION.

```
  37.0400
 704.3000
    .0376
 --------
 741.3776
```

EXPLANATION.—We write the numbers so that units of the same order stand in the same column; this brings the decimal points also in the same column. We commence at the right and add as in integers; bring down the decimal point and place it beneath, at the left of tenths' place.

RULE.

Write the decimals so that units of the same order shall stand in the same column and add as in simple numbers; and place the decimal point in the amount under the decimal points in the numbers added.

EXAMPLES.

(*1.*)	(*2.*)	(*3.*)	(*4.*)
.314	.23	.318	8.314
.2121	.375	.2647	2.002
.034	.4143	.03	3.01

(*5.*)	(*6.*)	(*7.*)	(*8.*)
3.002	1.814	274.3	35.7
14.07	2.3006	71.07	4.112
17.0143	4.05	80.013	19.
3.2	7.1	40.7112	9.0035

9. Add .7, .07, .175, and 2.0325.

10. Add 17.215, 3.056. .009, and 2.079.

11. Add 351.8, 45.001, 2.7169, and 34.21.

12. Add 3, 29.157, 8.116, and 123.49.

13. Add 25.29, 367.5, 2.091, and 55.

14. Add 21.03, 34.72, 5.005, and 25.1.

15. Add 31.91, 21.375, 4.03, and .005.

16. Add 3.042, 8.7562, 0.1437, and 0.021.

PRACTICAL PROBLEMS.

1. A man bought a horse for $203.75, a buggy for $305.50, a harness for $43.25, and a whip for $3.75; what was the cost of the whole? *Ans.* $556.25.

2. A farm consists of 33.125 acres of woodland, 24.876 acres of pasture land, 14.886 acres of meadow, and 13.888 acres of plough land; how many acres in the farm?
Ans. 86.775 acres.

3. A farmer sold a pair of horses for $287.755, a yoke of oxen for $173.865, and 7 cows for $213.825; what did he receive for them all?

4. A man bought a lot for $874.75, built a house on it that cost $5,843.795, a barn that cost $563.254, and then expended $2,811.345 for furniture; what was the cost of the whole?

What is addition of fractions? Give the rule for addition of *decimal fractions.*

SUBTRACTION OF DECIMALS.

147. Subtraction of Decimals is the operation of finding the difference between two decimal numbers.

1. A boy having 7 tenths of a dollar, paid 2 tenths for a trip to the park; how much had he left?

$\frac{2}{10}$ from $\frac{7}{10}$ leaves $\frac{5}{10}$, which expressed by means of the decimal point is .5.

2. From $\frac{25}{100}$ subtract $\frac{11}{100}$. $\frac{25}{100} - \frac{11}{100} = \frac{13}{100} = 13.$

3. From .8 subtract .3. $.8 - .3 = .5.$

4. From .78 subtract .5.

$$\frac{5}{10} = \frac{50}{100};\quad .78 - .50 = .28.$$

5. From 56.403 subtract 18.6.

ILLUSTRATION.

$$\begin{array}{r} 56.403 \\ \underline{18.6} \\ 37.803 \end{array}$$

EXPLANATION. — We write the subtrahend under the minuend, so that units of the same order shall stand in the same column; this will bring the decimal points in the same column. We then subtract as in simple numbers. Hence the following

RULE.

Write the subtrahend under the minuend so that units of the same order shall stand in the same column; then subtract as in simple numbers and place the decimal point in the remainder under the decimal points of the subtrahend and minuend.

EXAMPLES.

(*1.*)	(*2.*)	(*3.*)	(*4.*)
4.302	15.005	4.5301	5.6747
2.109	3.114	2.107	0.0328
2.193	11.891	2.4231	5.6419

(5.)	*(6.)*	*(7.)*	*(8.)*
37.290	8.41	102.	210.
18.143	3.987	3.884	.0014
19.147	4.423	98.116	209.9986

9. From 18.47 take 2.031.

10. From 13.81 take 8.492.

11. From 11.1156 subtract 2.0301.

12. From 1.805 subtract 0.0184.

13. 4.443—3.999=?

14. 8.123—6.015=?

15. 17.41—14.14=?

16. 120—25.75=?

17. 376.403 take 143.709.

18. 1,334.5 take 20.7362.

19. 8.0307 take 0.087.

20. 92.92 take 29.29.

PRACTICAL PROBLEMS.

1. A man had $18.75, but spent $4.755; how much had he left? *Ans.* $13.995.

2. A farmer bought groceries to the amount of $117.743, of which he paid $93.817 in oats, and the remainder in cash; how much did he pay in cash? *Ans.* $23.926.

3. A dealer bought goods for $8,743.85, and sold them for $11,342.81; how much did he gain?

4. Two fields contain 87.3142 acres, and the smaller one contains 32.8954 acres; what does the larger one contain?

5. From a hogshead of sugar weighing 993.142 *lbs.*, 418.387 *lbs.* were sold; how much remained?

TEST QUESTIONS.

What is subtraction of decimals? What kind of fractions can be subtracted? Give the rule for subtraction of decimals. What is the minuend? What the subtrahend? What is the result in subtraction?

MULTIPLICATION OF DECIMALS.

148. ***Multiplication of decimals is the operation of finding the product of two decimals.***

1. Multiply $\frac{1}{10}$ by 3.

$\frac{1}{10} \times 3 = \frac{3}{10}$ expressed by means of the decimal point, .3.

2. Multiply $\frac{2}{10}$ by $\frac{3}{10}$, $\frac{2}{10} \times \frac{3}{10} = \frac{3}{100} = .03$.

3. Multiply $\frac{3}{10}$ by $\frac{3}{100}$, $\frac{3}{10} \times \frac{3}{100} = \frac{9}{1000} = .009$.

4. Multiply $\frac{4}{10}$ by $\frac{2}{1000}$, $\frac{4}{10} \times \frac{2}{1000} = \frac{8}{10000} = .0008$.

If *tenths* are multiplied by *units*, the product is *tenths*.

If *tenths* are multiplied by *tenths*, the product is *hundredths*.

If *tenths* are multiplied by *hundredths*, the product is *thousandths*.

If *tenths* are multiplied by *thousandths*, the product is *ten thousandths*.

The denominator of the product will therefore contain as many ciphers as there are ciphers in the denominators of the factors. If the decimal be expressed by means of the decimal point, the number of decimal figures is still determined by the number of ciphers in the denominator; hence, the number of decimal figures in the product must equal the number of decimal figures in the factors.

5. Multiply .5 by 7.

Since tenths multiplied by units give tenths, .5 × 7 units = 35 tenths = 3.5.

6. Multiply .3 by .2.

Since tenths multiplied by tenths give hundredths, .3 × .2 = .06.

7. Multiply .5 by .09.

Since tenths multiplied by hundredths give thousandths, .5 × .09 = .045.

PRINCIPLE.—*There will be as many decimal places in the product as there are in the factors.*

RULE.

Multiply as in integers, and point off from the right of the product, as many decimal figures as there are in both factors.

Multiply the following

1. 4.7 by 3.2.
2. 21.2 by 0.12.
3. 17.4 by 2.3.
4. 3.18 by .34.
5. 3.51 by .75.
6. 4.81 by 8.2.
7. .03 by .12.
8. 3.12 by 2.4.
9. 86.28 by 2.5.
10. 7.81 by 3.25.
11. 1.057 by 1.032.
12. 3.33 by 2.22.
13. 8.19 by .034.
14. 0.143 by .12.
15. 402 by 0.43.
16. 99 by .25.
17. .845 by 16.
18. 175.6 by 3.24.
19. 38.42 by 3.27.
20. 56.57 by 12.35.
21. 387.2 by 8.143.

To multiply a decimal or a mixed decimal by 10, 100, 1000, etc., *move the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand if necessary.*

Multiply

22. .78 by 10.
23. 8.42 by 100.
24. .473 by 1000.
25. 6.5 by 100.

PRACTICAL PROBLEMS.

1. If a person earns $3.5 in 1 day, how much will he earn in 4.3 days? *Ans.* $15.05.

2. What will 36.75 *lbs.* of butter cost at 19.75 cents per *lb.*? *Ans.* $7.2581.

3. What will 17.3 tons of coal cost, at $7.75 per ton?

4. If a boat sails 7.35 miles in 1 hour, how far will she sail in 3.12 hours? How far in 5.25 hours?

5. If a barrel of flour weighs 203.5 *lbs.*, what will 7.33 barrels weigh? What will 8.05 barrels weigh?

6. What will 3.8 acres of land cost, at $55.72 per acre?

7. A merchant sold 7.3 *yds.* of cloth at $2.32 per yard; what did he receive for it?

8. There are 5.5 yards in 1 rod; how many yards are there in 71.24 rods? In 84.05 rods?

9. What is the cost of 51.3 gallons of wine, at $3.125 per gallon? What at $4.25 a gallon?

10. Find the value of 7.1 *lbs.* of meat, at 16.21 cents a pound. Find the value of 15.5 *lbs.*

11. If a man travels at the rate of 18.7 miles per day, how far will he travel in 7.11 days?

12. What will 8.5 *lbs.* of beef cost, at 22.5 cents a pound? What will 20.25 *lbs.* cost?

TEST QUESTIONS.

If tenths are multiplied by units, what denomination is the product? If tenths are multiplied by tenths, what is the product? If tenths are multiplied by hundredths, what is the product? If tenths are multiplied by thousandths, what is the product? Give the rule for multiplication of decimals. How do you multiply by 10, 100, 1,000, etc.

DIVISION OF DECIMALS.

149. Division of Decimals is the operation of finding the quotient of one decimal by another.

1. Divide 5 apples equally among 5 boys.

$$5 \div 5 = 1 \text{ (the fifth of 5).}$$

2. Divide .5 of an apple equally among 5 boys.

$$.5 \div 5 = .1 \text{ (the fifth of .5).}$$

3. Divide .05 of a dollar equally among 5 boys.

$$.05 \div 5 = .01 \text{ (the fifth of .05).}$$

4. Divide .005 by 5.

$$.005 \div 5 = .001 \text{ (the fifth of .005).}$$

5. Divide .0005 by 5.

$$.0005 \div 5 = .0001 \text{ (the fifth of .0005).}$$

6. Divide .5 by .5.

Reduce the decimal to the form of a common fraction.

$$.5 \div .5 = \tfrac{5}{10} \div \tfrac{5}{10} = \tfrac{5}{10} \times \tfrac{10}{5} = 1.$$

7. Divide .05 by .5.

$$.05 \div .5 = \tfrac{5}{100} \div \tfrac{5}{10} = \tfrac{5}{100} \times \tfrac{10}{5} = \tfrac{1}{10} = .1.$$

8. Divide .005 by .5.

$$.005 \div .5 = \tfrac{5}{1000} \div \tfrac{5}{10} = \tfrac{5}{1000} \times \tfrac{10}{5} = .01.$$

9. Divide .0005 by .5.

$$.0005 \times .5 = \tfrac{5}{10000} \div \tfrac{5}{10} = \tfrac{5}{10000} \times \tfrac{10}{5} = \tfrac{1}{1000} = .001.$$

10. Divide .05 by .05. $.05 \div .05 = \tfrac{5}{100} \div \tfrac{5}{100} = 1.$

From the nature of decimals as shown in these examples we have the following **Principles**:

1. *The dividend contains as many decimal places as the divisor and quotient together.*

2. *There must be as many decimal places in the quotient as the number of places in the dividend exceeds the number in the divisor.*

Let it be required to divide 1.38483 by 60.21.

ILLUSTRATION.

60.21)1.38483(.023
1 2042
18063
18063

EXPLANATION.—We divide as in integers, obtaining 23 as the result. We have now only to place the decimal point, and the work is done. Since the dividend has five decimal places, and the divisor two, the quotient should have three decimal places. As there are but two figures in the result, we supply the deficiency by prefixing a cipher, and place the decimal point at the left. Hence the following

RULE.

Divide as in integral numbers, and point off in the quotient, from the right hand, as many places for decimals as the number of decimal places in the dividend exceeds that in the divisor; and if there are not so many places, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide .74 by .25.
2. Divide 3.74 by .25.
3. Divide .008 by .5.
4. Divide 7.74 by 4.8.
5. Divide 2.56 by .0032.
6. Divide 3.1 by .25.
7. Divide 79.1 by .125.
8. Divide .12 by 1.6.
9. Divide .8 by 3.2.
10. Divide 8.25 by 25.

If the number of decimal places in the dividend is less than the number in the divisor, make them equal by annexing decimal ciphers to the dividend, and then divide.

11. Divide 6 by .003.
12. Divide 28.475 by .0005.

To divide by 10, 100, etc., we move the decimal point as many places to the left as there are ciphers in the divisor.

13. Divide 37.41 by 10.
14. Divide 341.6 by 100.
15. Divide 843.7 by 1000.
16. Divide 83.86 by 100.

TEST QUESTIONS.

If tenths are divided by tenths, how many decimal places will there be in the quotient? How many, if hundredths are divided by tenths? How many, if thousandths are divided by tenths? How many, if thousandths are divided by hundredths? Give the rule for division of decimals. State the three principles growing out of the nature of decimals. If the number of decimal orders in the dividend is less than those in the divisor, what must be done? How do you divide by 10, 100, 1000, etc.?

REDUCTIONS.

150. *To reduce decimals to common fractions.*

Reduce .75 to a common fraction.

ILLUSTRATION.

$.75 = \frac{75}{100} = \frac{3}{4}$

EXPLANATION.—We write the decimal in the form of a common fraction; and reduce it to its lowest terms. Hence, decimals may be reduced to common fractions by the following

RULE.

Express the decimal in the form of a common fraction, and reduce it to its lowest terms.

EXAMPLES.

Reduce the following decimals to common fractions:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
.032.	3.112.	7.002.	70.03.	.00012.

151. *To reduce common fractions to decimals.*

Reduce $\frac{3}{4}$ to a decimal fraction.

ILLUSTRATION.

$$\begin{array}{r} 4\,\overline{)\,3.00} \\ \hline .75 \end{array}$$

EXPLANATION.—$\frac{3}{4}$ is 3 divided by 4. If we annex one decimal cipher to 3, we obtain $3.0 = \frac{30}{10}$. This we divide by 4, and obtain the quotient $\frac{7}{10}$ and a remainder $\frac{2}{10}$. We write the quotient under the dividend, annex a cipher to the remainder 2, making $\frac{20}{100}$; divide it by 4, and write the result $\frac{5}{100}$ in hundredths place in the quotient. Hence the

RULE.

Annex as many decimal ciphers to the numerator as may be desired; divide the result by the denominator and point off from the right of the quotient a number of decimal figures equal to the number of ciphers annexed.

EXAMPLES.

Reduce the following common fractions to decimals:

1. $\frac{7}{8}$. *3.* $\frac{3}{5}$. *5.* $\frac{5}{7}$. *7.* $\frac{17}{20}$.
2. $\frac{11}{16}$. *4.* $\frac{11}{25}$. *6.* $\frac{18}{23}$. *8.* $\frac{9}{11}$.

The exact value of some common fractions cannot be expressed in decimals, as in Example 5. In such cases, continue the division until you obtain three decimal figures in the quotient, and annex the sign + to the result to denote incompleteness.

Reduce the following fractions to decimals:

9. $\frac{7}{11}$. *11.* $\frac{4}{17}$. *13.* $18\frac{2}{13}$. *15.* $\frac{1}{2}$ of $\frac{3}{7}$.
10. $\frac{9}{13}$. *12.* $6\frac{3}{16}$. *14.* $10\frac{4}{11}$. *16.* $\frac{15}{17}$.

NOTE.—If the number of figures in the quotient is less than the number of ciphers annexed, prefix the requisite number of ciphers.

PRACTICAL PROBLEMS.

1. If 3.5 bushels of wheat cost $4.375, what does 1 bushel cost? What do 2 bushels cost?

2. If 3.7 barrels of apples cost $20.35, what does 1 barrel cost? What do $2\frac{1}{2}$ barrels cost?

3. If a boat sails 34.78 miles in 3.7 hours, how far does she sail in 1 hour? How far in 2 hours?

4. If 18.2 *lbs.* of butter cost $6.825, what does 1 pound cost? What do 2 *lbs.* cost? What 5 *lbs.*?

5. What is the sum of 4.1 *acres*, 3.72 *acres*, 4.82 *acres*, and 8.15 *acres*? $5.3 + 6.05 + 7.005 + 3.06 = ?$

6. If a man travels 34.2 miles the first day, 19.9 miles the second day, 18.87 miles the third day, and 19.7 miles the fourth day, how far does he travel altogether?

7. A grocer bought 15.4 *lbs.* of butter at 37.5 cents a pound, and sold the lot for $7.225; how much did he gain by the bargain?

8. A. and B. start from the same point and travel in opposite directions; A. travels at the rate of 17.4 miles a day, and B. at the rate of 16.9 miles a day; how far apart are they at the end of 4.7 days?

9. If 6.5 *lbs.* of sugar costs 74.75 cents, what does 1 *lb.* cost? What do 3.7 *lbs.* cost?

10. If two persons are 38 miles apart, and start toward each other, the first at the rate of 3.5 miles per hour, and the second at the rate of 4.1 miles per hour, how long before they will meet?

REVIEW QUESTIONS.

What is a decimal unit? What is a decimal fraction? Define a decimal point. In what two ways may a decimal be written? Explain the method of reading a decimal. Repeat the numeration table from units to millions, and from units to millionths. State the three principles of decimals. Give the rule for reducing decimal fractions to common fractions. Give the rule for reducing common fractions to decimals. Reduce $\frac{3}{5}$ to a decimal. What is the rule for addition of decimals? For subtraction of decimals? What is the rule for multiplication of decimals? For division of decimals?

NOTE.—For more extended and thorough treatment of decimal fractions, use Davies and Peck's "Complete Arithmetic."

CURRENCY.

DEFINITIONS.

152. Currency is anything that circulates as lawful money.

153. Money is the measure of the value of things, used as a medium of trade.

154. Specie or coin is metal stamped with a die to give it a legal value, and authorized by government to be used as money.

155. A mint is a place in which the coin is manufactured.

156. Paper money consists of bills and notes authorized by government to circulate as money.

157. A Denominate Number is one whose unit is named; as, 3 *pounds*, 4 *feet*, 5 *dollars.*

158. A **simple denominate number** is a unit or collection of units of the same denomination; as, 1 *lb.*, $3, 8 *ft.*, etc.

159. A **compound number** is a collection of units of the same nature, but of different denominations; as, 2 *feet and* 4 *inches;* 3 *dollars and* 5 *cents;* 5 *pounds and* 8 *ounces.*

160. The **scale of a compound number** is a succession of numbers showing how many times the unit of each denomination is contained in the unit of the next higher denomination. Thus, in the table of English money, 4 farthings make 1 penny, 12 pence 1 shilling, 20 shillings 1 pound. The scale is 4, 12, 20. The scale of the United States currency is 10, 10, 10, 10, etc.

The first is *varying;* the second is *uniform.*

UNITED STATES CURRENCY.

161. The currency of the United States is a decimal currency.

TABLE.

10 mills (*m.*)	make	1 cent	*ct.*
10 cents	"	1 dime	*d.*
10 dimes	"	1 dollar	$.
10 dollars	"	1 eagle	*E.*

The coin of the United States consists of gold, silver, nickel and bronze.

Gold.—The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

Silver.—The trade-dollar, dollar, half-dollar, quarter-dollar, ten-cent piece, and five-cent piece.

Nickel.—Five-cent and three-cent pieces.

Bronze.—The one-cent and two-cent pieces.

Exercises for Oral Work.

1. How many mills in 1 cent? In 4 cents? In 8 cents?

2. How many cents in 1 dime? In 5 dimes? 6 dimes?

3. How many dimes in 1 dollar? In 8 dollars? In $10?

4. How many cents in 20 mills? In 30 mills? In 40?

5. How many dimes in 50 cents? In 80 cts.? In 90 *cts.*?

6. How many dollars in 40 dimes? In 60 dimes?

7. How many dimes and cents in 25 cents? In 75 *cts.*?

8. How many cents in 5 dollars? In 50 dollars?

9. How many dollars in 300 cents? In 2,000 cents?

10. How many cents in 15 dollars? In 18 dollars?

Expressions for United States currency are usually written in the form of a mixed decimal, the *dollar* being taken as the primary unit. Thus, the expression 3 *eagles, 2 dollars,* 5 *dimes,* 4 *cents,* and 3 *mills* is written $32.543, and read 32 *dollars,* 54 and 3 *tenths cents.*

The terms *dime* and *eagle* are but little used; the term *mill* is seldom employed, except in official reports and in laying taxes. In business operations mills are generally expressed as fractions of a cent. Since dimes are tenths of a dollar, when the number of cents is less than ten, we write a cipher in the place of tenths. Thus, $2.05.

Expressions for United States currency are usually read in dollars and cents.

Read

$31.25. $123.04. $108.036. $100.042. $88.888.

Application of the Rules for Decimals to U. S. Currency.

EXAMPLES FOR WRITTEN WORK.

1. Write decimally 3 *eagles,* 4 *dollars,* 3 *dimes,* 5 *cents,* and 2 *mills.*

2. Write 14 *eagles,* 4 *dimes,* 5 *cents,* and 7 mills.

3. Write decimally 86 *eagles*, 3 *dollars*, 2 *cents*, and 3 *m.*

4. Write decimally 70 *eagles*, 4 *dollars*, and 8 *cents.*

5. Add \$16.54, \$13.43, \$81.415, and \$9.607.

6. Add \$3,814, \$40.60, \$31.875, and \$118.436.

7. Add \$8.14, \$32.415, \$4.675, and \$31.843.

8. Add \$0.17, \$8.477, \$30.303, and \$8.888.

9. From \$14.62 subtract \$1.89. \$13.43—\$9.235 = ?

10. Find the difference between \$25.873 and \$19.984.

11. A man bought a box of raisins for \$3.375, a box of candles for \$4.62, and 75 *lbs.* of sugar for \$9.465; what did he pay for the whole?

12. A person bought a hat for \$4.75, a coat for \$19.65, a pair of boots for \$8.44, and an umbrella for \$1.69; what did they all cost him?

13. A bookseller sold a dictionary for \$8.25, 6 readers for \$7.44, 5 arithmetics for \$4.75, and 20 spellers for \$4.75; what did he receive for them all?

14. A dealer bought cloth for \$18.42, and sold it again for \$27.105; what did he gain?

15. A boy had \$4.75 at the beginning of the week, and earned during the week \$5.92; if he spends \$6.18, how much will he then have?

16. A farmer sold a horse for \$187.25, two cows for \$84.90, and 40 bushels of oats for \$22.46, for which he received a wagon worth \$213.26, and the balance in cash; how much cash did he receive?

17. Multiply \$4.75 by 9.
18. \$12.625 × 13 = ?
19. \$88.775 × 27 = ?
20. \$84.35 × 12.2 = ?
21. 118.26 ÷ 9 = ?
22. \$1,220.03 ÷ 14 = ?
23. \$2,022.02 ÷ \$77.77 = ?
24. \$2.895.0438 ÷ \$86.86 = ?

CANADA CURRENCY.

162. The currency of the Dominion of Canada is decimal; it is reckoned in *dollars* and *cents*.

The coin of Canada is silver and bronze.

FRENCH CURRENCY.

The coin of France is gold, silver and bronze.

TABLE.

10 centimes (*ct.*)	make	1 decime, *dc.*
10 decimes	"	1 franc, *fr.*

The primary unit is 1 franc; its value is $0.193.

ENGLISH MONEY.

163. This is the national currency of Great Britain.

The primary unit is 1 *pound sterling.*

TABLE.

4 farthings (*far.*, or *qr.*)	make	1 penny, *d.*
12 pence	"	1 shilling, *s.*
20 shillings	"	1 pound sterling, £.
21 shillings	"	1 guinea, *g.*

The sign £ is usually written before the number to which it applies.

The coin of Great Britain is gold, silver and copper.

The pound is the unit of the *money of account*, and the coin representing it is called a sovereign.

The value of the sovereign is $4.8665 U. S.

2 shillings make 1 florin, *fl.*
5 shillings make 1 crown, cr.

EXAMPLES.

1. In £5 how many shillings? How many in £8?

2. How many pence in 18*s.*? How many in 11*s.*?

3. How many pounds in 40*s.*? How many in 60*s.*?

4. How many dollars in 10 *sov.*? How many in 8 *sov.*?

5. How many *sov.* in 100*s.*? How many in 120*s*?

6. In 6 francs how many centimes?

7. In 5 francs how many decimes?

8. In 250 centimes how many francs?

9. In 140 francs how many U. S. dollars?

10. In $20 how many francs?

11. How many U. S. dollars in £4?

12. How many U. S. dollars in 60*s.*?

13. How many U. S. dollars will it require to pay a draft from London amounting to 258 pounds sterling?

14. I wish to pay a merchant in Paris 1250 francs; how many U. S. dollars will it require?

15. How many U. S. dollars and cents in £3 5*s.* 6*d.*

What is currency? What is money? What specie? What, a mint? Of what does paper money consist? What is the currency of the United States? Recite the table. How are expressions for U. S. currency usually written? When the number of cents is less than 10, what is written in place of tenths of cents? In what denominations are expressions of U. S. currency usually read? What is the currency of Canada? How reckoned? What is the coin of Canada? Of what does the coin of the U. S. consist? Name the denominations of the gold coin. Of the silver coin. Of nickel. Of bronze. What is the primary unit in the U. S. and Canada currency? Recite the table of French currency. What is the primary unit? What is its value in U. S. currency? Recite the table of English money. What is the primary unit? What is its value in U. S. currency? How is the pound sterling used? What coin takes its place in the currency of England? What is the value of the *sovereign in U. S.* currency?

METRIC SYSTEM.

164. The **Metric System** is a system of weights and measures based on a primary unit of length called a **Meter.** *The scale is decimal.*

The **Meter** is one *ten-millionth* part of the distance from the equator to the north pole. It is nearly equal to 39.37 inches.

MEASUEE OF LENGTH.

165. The primary unit is the **meter.**

TABLE.

10 millimeters (*mm.*)	make	1 centimeter (*cm.*)
10 centimeters	"	1 decimeter (*dm.*)
10 decimeters	"	1 meter (*m.*)
10 meters	"	1 decameter (*decam.*)
10 decameters	"	1 hectometer (*hectom.*)
10 hectometers	"	1 kilometer (*kilom.*)
10 kilometers	"	1 myriameter (*myram.*)

NOTE.—If possible show the class a *meter*, and explain its subdivisions. If none is at hand, construct one from the Decimeter on the 158th page.

EXERCISES AND EXAMPLES.

1. Measure your desk, and tell how many meters, or meters and decimeters long it is. How many wide?

2. Measure the door, both its height and width.

3. How many centimeters long is your arithmetic? *How many wide?*

4. How many milimeters wide is the Decimeter illustrated on this page?

5. How many decimeters in 12 meters?

6. How many centimeters in 20 millimeters? How many in 50?

7. Change 25 decameters to meters and decimals of a meter.

8. Change 225 hectometers to meters and decimals of a meter.

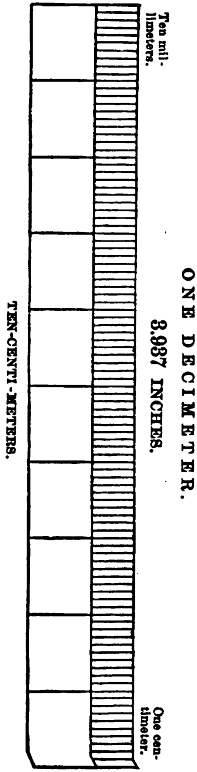
To Write Numbers in the Metric System.

Write the meters in the units place, decameters in tens place, hectometers in hundreds place, the kilometers in thousands place, myriameters in the ten thousands place, decimeters in the tenths place, centimeters in the hundredths place, and millimeters in the thousandths place.

To Read Numbers in the Metric System.

We may read a number in the Metric system in terms of all its units, or in terms of any one of them.

The expression 24.25 meters may be read 2 decameters, 4 meters, 2 decimeters, and 5 centimeters; but it is usually read 24 and *twenty-five hundredths* meters.



Exercises in Reading Numbers.

Read the following:

16 *m.*; 50 *dm.*; 75 *cm.*; 14 *mm.*; 17 *decam.*; 28 *kilom.*

MEASURE OF SURFACE.

166. The primary unit is the **Square Meter.**

TABLE.

100 sq. millimeters (*sq. mm.*) make 1 sq. centimeter, (*sq. cm.*)
100 sq. centimeters make 1 sq. decimeter, (*sq. dm.*)
100 sq. decimeters " 1 sq. meter, (*sq. m.*)
100 sq. meters " 1 are, (*a.*)
100 ares " 1 hectare, (*ha.*)

For *land measure* the primary unit is the **are** (pronounced *ar*).

MEASURES OF VOLUME.

167. The primary unit is the **stere** (pronounced *stair*); it is a **cubic meter.**

TABLE.

1000 cu. centimeters (*cu. cm.*) make 1 cu. decimeter, (*cu. dm.*)
1000 cu. decimeters make 1 stere.

For Wood Measure.

10 decisteres (*d. st.*) make 1 stere, (*st.*)
10 steres " 1 decastere, (*dec. st.*)

The primary unit is the stere = .2759 cords.

MEASURES OF CAPACITY.

The primary unit is the liter (*leeter*) = 61.026 cu. in.

TABLE.

10 centiliters (*cl.*)	make	1 deciliter, (*dl.*)
10 deciliters	"	1 liter, (*l.*)
10 liters	"	1 decaliter, (*dec. l.*)
10 decaliters	"	1 hectoliter, (*hec. l.*)
10 hectoliters	"	1 kiloliter, or stere, (*k. l.*)

MEASURE OF WEIGHTS.

168. The primary unit is the gram. It is the weight of a cubic centimeter of distilled water at 39° Fah.

TABLE.

10 milligrams (*m.g.*)	make	1 centigram, (*c. g.*)
10 centigrams	"	1 decigram, (*d. g.*)
10 decigrams	"	1 gram, (*g.*)
10 grams	"	1 decagram, (*dec. g.*)
10 decagrams	"	1 hectogram, (*hec. g.*)
10 hectograms	"	1 kilogram, (*kil. g.*)

Small weights are expressed in milligrams, and large ones in kilograms. The kilogram is the weight of a liter of water at 39° Fah.

EXAMPLES.

Change the following to meters:

(*1.*)	(*2.*)	(*3.*)	(*4.*)
30 dm.	800 *cm.*	12000 mm.	150 *decam.*

(5.)	*(6.)*	*(7.)*
200 *hectom.*	5,000 *kilom.*	30,000 *myriam.*

Change the following to square meters:

(8.)	*(9.)*	*(10.)*	*(11.)*	*(12.)*
200 *sq. dm.*	3000 *sq. cm.*	6000 *sq. mm.*	15 *a.*	27 *ha.*

Reduce the following to steres:

(13.)	*(14.)*	*(15.)*	*(16.)*	*(17.)*
5000 *cu. dm.*	15 *dec. st.*	3000 *cu. dm.*	400 *d. st.*	20 *dec. st.*

Reduce the following to liters:

(18.)	*(19.)*	*(20.)*	*(21.)*	*(22.)*
500 *cl.*	120 *dl.*	20 *dec. l.*	300 *hec. l.*	4000 *k. l.*

Change

(23.)	*(24.)*	*(25.)*
20 *d. g.* to *dec. g.*	500 *hec. g.* to *g.*	400 *m. g.* to *c. g.*

(26.)	*(27.)*
6,000 *g.* to *kil. g.*	5 *hec. g.* to *g.*

28. Express in kilograms and decimals of a kilogram.

3 *kil. g.* 25 *hec. g.* 15 *dec. g.* 6 *g.*

On what is the Metric system based? What is the scale in the notation of the Metric system? The meter is what part of the distance from the equator to the north pole? What is its length in inches? Recite the table of length. The meter is $\frac{1}{10000000}$ part of what? How are numbers written in the Metric system? How may a number be read in the Metric system? Write the abbreviations of the denominations in the table of length. Recite the table of measure of surface. Draw a square decimeter on the blackboard. Draw a square centimeter. How many square millimeters in the square centimeter?

BUSINESS OPERATIONS.

TERMS USED IN BUSINESS TRANSACTIONS.

169. The cost of a thing is the *value in money* paid for it.

170. *To find the cost of any number of things when we know the cost of one.*

1. What is the cost of 40 bushels of oats at 57 cents a bushel?

ILLUSTRATION.

```
 $0.57
    40
$22.80
```

EXPLANATION.—Since 1 bushel costs $0.57, 40 bushels will cost 40 times $0.57 = $22.80.

RULE.

Multiply the cost of one thing by the number of things.

EXAMPLES.

2. What is the cost of 5 yards of cloth at $5.50 a yard?

3. Find the cost of 12½ cords of wood at $7.50 a cord.

4. Find the cost of 56 *lbs.* of pork at 9¼ *cts.* a pound.

Since it is not material which factor is used as the multiplier, in examples like the last, for the sake of convenience, we multiply by the smaller number, and observe the rule for pointing off decimals.

5. Find the cost of 32½ *yds.* of ribbon at 37½ *cts.* a yard.

6. Find the cost of 43¼ *lbs.* of tea at 92½ *cts.* a pound.

7. A cask of wine contains 42¾ gallons, and is worth *$2.13 per gallon*; what is the value of the whole?

171. *To find the cost of one thing, when we know the entire cost, and the number of things.*

1. If 50 oranges cost $2.25, what do they cost apiece?

Since 50 oranges cost $2.25, one orange costs $\frac{1}{50}$ of $2.25 = $0.045.

RULE.

Divide the entire cost by the number of things.

EXAMPLES.

2. Jane bought 47 *yds.* of calico for $6.34½; what did it cost per yard? *Ans.* 13½ *cents.*

3. If 63½ bushels of potatoes cost $35.24¼, what is the cost of 1 bushel, and what is the cost of 8 bushels?

4. Find the cost of 1 *lb.* of tea, when 37½ *lbs.* cost $28.12½. Find the cost of 3 *lbs.*

5. If 44 *yds.* of linen cost $27.50, what is the cost of 1 *yd.*? Of 16½ *yds.*? Of 26 *yds.*?

6. If 48 *lbs.* of grapes cost $15.36, what will 13 *lbs.* cost?

7. How many *lbs.* of sugar, at 14 *cts.* a pound, can be bought for 56 *lbs.* of butter, at 31¼ *cts.* per *lb.*?

8. A farmer sold 8 cows for $345; what was the average value of each?

172. *To find the number of things when we know the entire cost, and the cost of one thing.*

1. How many pounds of sugar, at 12½ cents a pound, can be bought for $106.25?

SOLUTION. 106.25 ÷ .12½ = 850, therefore 850 lbs. can be bought.

RULE.

Divide the entire cost by the cost of one thing.

EXAMPLES.

2. How many yards of muslin, at 19½ cents a yard, can be bought for $22.815? How many for $50?

3. How many barrels of flour, at $12.44 a barrel, can be bought for $485.16? How many for $625?

4. If a man earns $18.75 per week, how many weeks will it take him to earn $323.43¾?

5. How many yards of linen, at 64½ cents a yard, can be bought for $200.595? How many for $500?

6. How many barrels of flour, at $7.50 a barrel, can be bought for $217.50? How many for $750?

7. If tea costs 85½ cents a pound, how many pounds can be bought for $117.56¼? How many for $235.12½?

8. How many bushels of potatoes, at 37½ cents a bushel, can be bought for $10.12½? How many for $30.37½?

173. *To find the cost of any number of things, when we know the cost of 100 or 1,000 things.*

1. What is the cost of 217 bananas, at $2.15 per hundred?

ILLUSTRATION.

$$\frac{\$2.15}{100} = \begin{array}{r} \$0.0215 \\ 217 \\ \hline 1505 \\ 215 \\ 430 \\ \hline \$4.6655 \end{array}$$

EXPLANATION.—Dividing $2.15 by 100, gives the cost of one banana; and multiplying the cost of one by 217, gives the cost of 217 bananas.

RULE.

Multiply the cost of 100 or 1,000 things by the number of things, and move the decimal point two places to the left when the cost of 100 is given, and three places when the cost of 1,000 is given.

EXAMPLES.

2. Find the cost of 1,622 bricks, at $6.25 per thousand.

3. What is the value of 874 *ft.* of scantling, at $3.50 per hundred?

4. Find the cost of 3,240 shingles, at $5 per thousand.

5. How much must I pay for 375 *lbs.* of beef, at $7.50 per hundred?

6. If oranges sell at $2.25 per hundred, how much must I pay for 288 oranges?

174. ***To find the cost of any number of pounds when we know the cost of a ton of 2,000 pounds.***

1. Find the cost of a load of coal weighing 2,400 pounds, at $8.64 a ton.

ILLUSTRATION.

```
2)8.64
  4.32
   2400
 ------
 172800
 864
 ---------
$10.368.00
```

Explanation.—Since $8.64 is the cost of 2,000 pounds, ½ of $8.64 or $4.32 is the cost of 1,000 pounds; 2400 pounds will cost $\frac{2400}{1000}$ times $4.32. We multiply by 2400, and move the decimal point three places to the left, which is equivalent to dividing the product by 1000.

RULE.

Multiply half the cost of 1 ton by the number of pounds, and move the decimal point three places to the left.

EXAMPLES.

2. What is the cost of 8,136 *lbs.* of coal, at $7 per ton?

3. Find the cost of 3,714 *lbs.* of steel. at $18.50 a ton.

4. Find the cost of 8,176 *lbs.* of straw, at $16 a ton.

5. Find the value of 8,488 *lbs.* of hay, at $14 per ton.

6. What is the cost of transportation of 3,635 *lbs.*, at the rate of $1.28 per ton?

7. Find the value of 70,432 *lbs.* of ore, at $2.50 per ton.

8. If building stone is worth $1 per ton, what will 88,311 *lbs.* cost?

ALIQUOT PARTS.

175. An **Aliquot Part** of a number is one of the equal parts, whether integral or fractional, into which the number can be divided.

The most important aliquot parts of a dollar are given in the following

TABLE.

50 *cts.* is $\frac{1}{2}$ of $1.	12½ *cts.* is $\frac{1}{8}$ of $1.
33⅓ *cts.* " $\frac{1}{3}$ of $1.	10 *cts.* " $\frac{1}{10}$ of $1.
25 *cts.* " $\frac{1}{4}$ of $1.	6¼ *cts.* " $\frac{1}{16}$ of $1.
20 *cts.* " $\frac{1}{5}$ of $1.	5 *cts.* " $\frac{1}{20}$ of $1.

176. To find the cost of any number of things when 1 thing costs an aliquot part of $1.

1. What is the cost of 60 *lbs.* of butter, at $33\frac{1}{3}$ *cts.* a pound?

Illustration.—At $1 a pound, 60 pounds will cost $60. Since the cost of 1 pound is $\frac{1}{3}$ of a dollar, the cost of 60 pounds will be $\frac{1}{3}$ of $60 or $20.

RULE.

Divide the number of things by the number of times that the price of one thing is contained in $1.

EXAMPLES.

2. What is the cost of 40 bushels of potatoes, at $33\frac{1}{3}$ cents a bushel?

3. Find the cost of 87 *yds.* of sheeting, at 25 cents a *yd.*

4. What is the cost of 147 *yds.* of calico, at 20 *cts.* a yard? What is the cost of 180 yards?

5. How much will 168 *lbs.* of sugar come to, at $12\frac{1}{2}$ *cts.* a pound? How much at $33\frac{1}{3}$ cents?

6. Find the cost of 64 bushels of oats, at $37\frac{1}{2}$ *cts.* a bushel.

The cost will be 3 times as much as though the price was $12\frac{1}{2}$ cents a bushel; hence it is $\frac{1}{8}$ of 64×3 or $24.

7. Find the cost of 76 bushels of rye, at 75 *cts.* a bushel. Find the cost of 225 bushels.

8. How much will 118 *lbs.* of soap cost, at $12\frac{1}{2}$ *cts.* a pound? How much, at $6\frac{1}{4}$ cents?

177. *To find the number of things when the entire cost is known, and the cost of one is an aliquot part of a dollar.*

1. How many bushels of apples at \$0.50 a bushel, can be bought for \$212 ?

ILLUSTRATION.	**EXPLANATION.**—Since $\$\frac{1}{2}$ will buy one bushel, \$212 will buy as many bushels as there are $\$\frac{1}{2}$ in \$212.
$\$212 \div \$\frac{1}{2} = 424$	

RULE.

Divide the entire cost by such a fraction as will express the cost of one as an aliquot part of a dollar.

EXAMPLES.

2. How many baskets of pears can be bought for \$10 at $33\frac{1}{3}$ cents a basket?

3. How many pineapples can be bought for \$22 at $12\frac{1}{2}$ cents each?

4. How many pounds of cheese can be bought for \$27 at \$0.25 a pound? How many for \$50?

5. How many quarts of peas can be bought for \$3 at \$0.20 a quart? How many for \$5?

6. How many bushels of oats, at 75 cents a bushel, can be bought for 45 bushels of wheat at \$2.25 a bushel?

What is currency? Illustrate. Repeat the table of United States currency. In what terms do we write United States currency? By what rules do we operate on United States currency? How do you find the cost of any number of things when you know the cost of one? What is cost? How do you find the cost of one when you know the entire cost and the number of things? How do you find the number of things when you know the entire cost, and the cost of one? How do you find the cost of any number of things sold by the hundred or thousand? How do you find the cost of any number of pounds when you know the price per ton? What is an aliquot part of a number? How do you find the cost of any number of articles when the price of one article is an aliquot part of a dollar?

BILLS AND ACCOUNTS.

178. A **Bill** is a written statement of goods sold, of services rendered, or of money paid.

179. An **Account** is a written statement of items of debit and credit.

180. A **Debtor** is a person who owes another.

181. A **Creditor** is a person to whom something is owed.

182. A **Bill is receipted** when the words "Received Payment" are written at the bottom, and the creditor's name is lawfully signed.

183. A **Footing** is the sum or amount of a bill.

184. **Credit** is what the creditor pays, or has paid. **Debit** is what the debtor owes.

185. The **Balance** is the difference between the footings of the debit and credit sides of an account.

186. Abbreviations used in Accounts.

@. At.
%. Account.
Am't. Amount.
Bal. Balance.
Co. Company.
Cr. Credit or Creditor.
Do. The Same.
Doz. Dozen.
Dr. Debit or Debtor.
No. Number.
Pd. Paid.
Per. By.
Rec'd. Received.
% Per cent.

187. ***To find the footing of a bill.***

Find the amount of each item and take the sum of the results.

Find the amount of the following bill of items:

New York, May 1, 1878.

Mr. James Spendthrift,

Bought of Benjamin Saveall,

16 pounds of Tea, @ $0.85 per pound		
20 " " Coffee, @ $0.15½ per pound . . .		
15 yards of Linen, @ $0.66 per yard		
Amount, . . .	$	

Received payment,

Benjamin Saveall.

Albany, June 2, 1878.

Mr. Jacob Johns,

Bought of Gideon Gould,

36 pounds of Sugar, at 9½ cents per pound . . .		
3 hogsheads of Molasses, 63 galls. each, @ 27 cents a gallon		
5 casks of Rice, 200 pounds each, @ 5 cts. per pound		
2 chests of Tea, 80 pounds each, @ 96 cts. per pound		
Total cost, . .	$	

Received payment, for Gideon Gould,

Charles Clark.

188. *To find the balance of an account.*

Find the difference of the debit and credit items of the account.

Find the balance of the following account:

A. T. Stewart & Co.

In account with A. L. Compton.

Dr. Cr.

1878											
Jan.	12	To 200 lbs. Butter @	$.30			Feb.	5	By 20 yds. Cloth @	$2.		
Feb.	10	" 900 lbs. Cheese "	.18			Mar.	8	" 25 prs, Gloves "	1.20		
"	15	" 400 lbs. Lard "	.12			Apr.	1	" 12 yds. Linen "	0.50		
Mar.	3	" 500 lbs. Tallow "	.16								
Apr.	4	" 80 doz. Eggs "	.25			Apr.	4	Balance			

COMPOUND NUMBERS

189. A **Compound Number,** as already defined, is a denominate number whose units are of the same kind, but of different denominations.

Denominate numbers are of the *same kind* when they can be expressed in terms of a common unit. Thus, 3 *shillings* and 5 *pence* are of the same kind, because they can both be expressed in *pence.*

If two denominate numbers are of the same kind, that which has the *greater unit* is said to be of the *higher denomination.* Thus, 3 shillings is of a higher denomination than 5 pence.

TABLES OF WEIGHTS.

TROY WEIGHT.

190. This is used in weighing gold, silver, and some kinds of precious stones.

The primary unit is 1 pound.

TABLE.

24 grains (*gr.*)	make	1 pennyweight .	*dwt.*
20 pennyweights	"	1 ounce . . .	*oz.*
12 ounces	"	1 pound Troy .	lb. Tr.

APOTHECARIES WEIGHT.

191. This table is used in mixing medicines.

The primary unit is 1 pound, the same as Troy weight.

TABLE.

20 grains (*gr.*)	make	1 scruple . .	℈
3 scruples	"	1 dram . . .	ʒ
8 drams	"	1 ounce . . .	℥
12 ounces	"	1 pound . .	℔

AVOIRDUPOIS WEIGHT.

192. This is used in weighing the ordinary articles of trade and commerce.

The primary unit is 1 pound, equal to 7000 grains Troy weight.

TABLE.

16 ounces (*oz.*)	make	1 pound . . .	*lb.*
25 pounds	"	1 quarter . . .	*qr.*
4 quarters	"	1 hundred . .	*cwt.*
20 hundred	"	1 ton	*T.*

NOTE.—In weighing coarse articles liable to wastage, as coal at the mines, etc., it is customary to call 112 lbs. a hundredweight, and 28 lbs. a quarter.

TABLE OF TIME.

193. Time is a measured portion of duration. Its primary unit is one solar day.

An **astronomical year** is the time required for the earth to perform one revolution round the sun; but this period does not contain an exact number of days; hence, for civil purposes, an artificial year is adopted. The ***artificial* or *civil* year** has sometimes 365 days, and sometimes *366 days*, so distributed that, after a long period, the average length of the civil year is nearly equal to that of the astronomical year.

Every year divisible by 4 (except *centennial* years not divisible by 400) are leap years, all other years are common years.

TABLE.

60 seconds (*sec.*)	make	1 minute . . .	*min.*
60 minutes	"	1 hour	*hr.*
24 hours	"	1 day	*da.*
7 days	"	1 week	*wk.*
365 days	"	1 common year .	*c. yr.*
366 days	"	1 leap year . .	*l. yr.*
100 years	"	1 century . . .	*C.*

The civil year is divided into 12 unequal parts called months. Their order, and the number of days in each, are shown in the table following:

TABLE OF MONTHS.

1. January . .	31 days,	7. July	30 days,
2. February .	28 "	8. August . . .	31 "
3. March . . .	31 "	9. September . .	30 "
4. April	30 "	10. October . . .	31 "
5. May	31 "	11. November . .	30 "
6. June	30 "	12. December . .	31 "

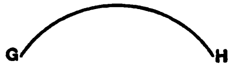
February has 28 days in common years, and 29 days in leap years.

MEASURES OF LENGTH.

194. Magnitude is anything that can be measured.

195. A **Line** is the magnitude of length without regard to thickness or breadth.

A curved line is one whose direction changes at every point; as GH.

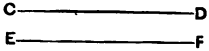


CURVED LINE.

A straight line is one whose direction does not change at any point; as AB.

STRAIGHT LINE.

Straight lines are parallel when they have the same direction; as CD and EF.



PARALLEL LINES.

The length of a line is the number of times it contains a given straight line taken as a unit.

LONG MEASURE.

196. This table is used in measuring distances and lines. The yard is the primary unit.

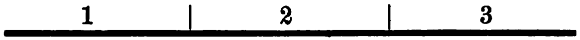


TABLE.

12	inches (*in.*)	make	1 foot	*ft.*
3	feet	"	1 yard	*yd.*
5½	yards	"	1 rod	*rd.*
40	rods	"	1 furlong	*fur.*
8	furlongs, or 320 rods	"	1 mile	*mi.*
3	miles	"	1 league	*lea.*

SURVEYORS' MEASURE.

197. Used in measuring land.

The unit is a Gunter's chain which is equal to 66 feet. It is divided into 100 equal parts called links.

TABLE.

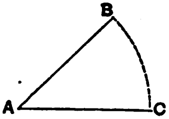
7.92	inches	make	1 link	*li.*
100	links, or 4 rods	"	1 chain	*ch.*
80	chains, or 320 rods	"	1 mile	*mi.*

MEASURES OF SURFACE.

DEFINITIONS.

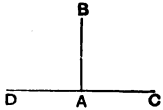
198. A **Surface** is a magnitude that has length and breadth, without thickness.

199. An **Angle** is the opening between two lines that meet at a point; as, BAC. The lines AB and AC are called **sides**, and the point A is called the **vertex** of the angle.



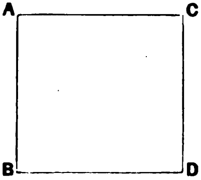
ANGLE. Fig. 2.

If the angle BAD is equal to the angle BAC, BA is **perpendicular** to DC, and the angles BAD and BAC are **right angles.**



RIGHT ANGLES. Fig. 1.

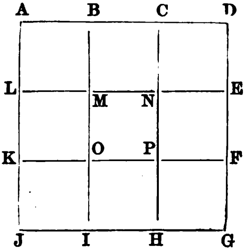
200. A **Square** is a plane figure bounded by four equal sides, and whose angles are right angles; as Fig. 3.



SQUARE. Fig. 3.

201. A **Rectangle** is a plane figure whose opposite sides are equal, and whose angles are right angles.

A **square foot** is a square whose sides are each equal to one foot; a **square yard** is a square whose sides are each equal to 1 yard.



SQUARE. Fig. 4.

The unit of measure of a surface is a square, one of whose sides is equal to the unit of length.

202. The **Area** of a surface is an expression for that surface in terms of a square unit. Figure 4 represents a square yard, each side of which is three feet long; the **area** is nine square feet.

The area of a square or rectangle is found by multiplying its length by its breadth.

SQUARE MEASURE.

Used for measuring surfaces.

TABLE.

144	square inches (*sq. in.*)	make	1 square foot	*sq. ft.*
9	square feet	"	1 square yard	*sq. yd.*
30¼	square yards	"	1 square rod	*sq. rod.*
160	square rods	"	1 acre	*A.*

LAND MEASURE.

This is used in finding the area of land.

TABLE.

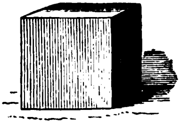
10,000	square links (*sq. li.*)	make	1 square chain	*sq. ch.*
10	square chains	"	1 acre	*A.*
640	acres	"	1 square mile	*sq. mi.*

In government surveys, a square mile is called a *section*, and 36 sections make a *township*.

MEASURE OF VOLUME AND CAPACITY.

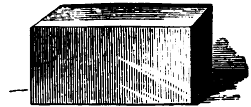
203. A **volume** or **solid** is a magnitude that has *length, breadth*, and *thickness* or height.

204. A **cube** is a volume or solid bounded by six equal squares. The bounding squares are called **faces.** The sides of the squares are called **edges** of the cube.



CUBE.

205. A **unit of volume** is a cube whose edges are equal to a unit of length.



PARALLELOPIPEDON.

CUBIC MEASURE.

206. This is used in measuring volumes or solids.

TABLE.

1728 cubic inches (*cu. in.*) make 1 cubic foot . *cu. ft.*
27 cubic feet " 1 cubic yard . *cu. yd.*

A cord of wood is a pile 4 ft. wide, 4 ft. high, and 8 ft. long. A foot in length from such a pile is a **cord foot.**

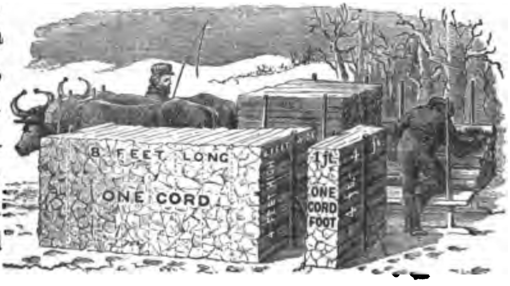


TABLE.

16 cubic feet (*cu. ft.*) make 1 cord foot . *C. ft.*
8 cord feet " 1 cord . . . *C.*

DRY MEASURE.

207. This is used in measuring grain, fruit, salt, *etc.*

The primary unit is 1 bushel. It is a cylindrical measure 18½ inches across, and 8 inches deep; containing 2,150⅖ cubic inches.

TABLE.

2 pints (*pt.*)	make	1 quart	*qt.*
8 quarts	"	1 peck	*pk.*
4 pecks	"	1 bushel	*bu.*

LIQUID MEASURE.

208. This is used in measuring liquids.

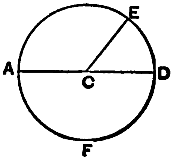
The primary unit is 1 gallon. It contains 231 cubic inches.

TABLE.

4 gills (*gi.*)	make	1 pint	*pt.*
2 pints	"	1 quart	*qt.*
4 quarts	"	1 gallon . . .	*gal.*
31½ gallons	"	1 barrel . . .	*bbl.*
2 barrels	"	1 hogshead . .	*hhd.*

ANGULAR MEASURE AND LONGITUDE.

DEFINITIONS.



CIRCLE.

209. A **Circle** is a portion of a plane bounded by a curved line every point of which is equally distant from a point within called the *centre;* as, AED. The bounding line is called the *circumference;* as, AED. Any part of the bounding line is called an *arc;* as, ED.

If a circumference is divided into 360 equal parts, each part is called a *degree.*

210. A **Diameter** is a straight line passing through the centre and terminating in the circumference; as, AD.

211. A **Radius** is a straight line drawn from the centre to any part of the circumference; as, CD, CE.

ANGULAR MEASURE.

212. The primary unit is the right angle. The ninetieth part of a right angle is a degree.

TABLE.

60	seconds (″)	make	1 minute . . .	′.
60	minutes	"	1 degree . . .	°.
90	degrees	"	1 right angle . .	*rt. a.*
360	degrees	"	1 circumference .	*cir.*

LONGITUDE.

Meridian Lines.

213. Longitude of a place is the distance east or west from a given meridian.

The earth revolves on its axis once in 24 hours.

The circumference of the earth is a circle, and, like other circles, is supposed to be divided into 360 equal parts called degrees (°).

The sun appears to pass

entirely round the earth, 360°, once in 24 hours; in 1 hour it passes $\frac{1}{24}$ of 360°, or 15° of longitude. Hence, for a difference of 15° of longitude there is a difference of 1 hour of time, for 1° of longitude there is a difference of $\frac{1}{15}$ of 1 hour of time, or 4 minutes; for 15′ of longitude there is a difference of 1 minute of time; for 15″ of longitude there is a difference of 1 second of time. It is noon, or 12 o'clock, at any place when the sun is on the meridian of that place; 15° west of that place it is 1 hour earlier, or 11 o'clock; 15° east it is 1 hour later, or 1 o'clock, etc. Hence the following

TABLE.

A difference of 15° of longitude makes a difference of 1 hour of time.

A difference of 15′ of longitude makes a difference of 1 minute of time.

A difference of 15″ of longitude makes a difference of 1 second of time.

REDUCTION.

214. Reduction is the process of changing the form of a number without altering its value. Thus, $25 may be expressed as 2500 cents, 500 cents may be expressed as $5.

215. Reduction Descending is the operation of changing a number from a higher to a lower denomination; as, dollars to cents.

216. Reduction Ascending is the operation of changing a number from a lower to a higher denomination; as, cents to dollars.

EXAMPLES IN REDUCTION DESCENDING.

FOR ORAL WORK.

217. *1.* How many feet in 3 yards? In 5 yards?

2. How many feet in 4 yards and 2 feet? 5 *yds.* 3 *ft.*?

3. Reduce 16 feet to inches. Reduce 15 yards to *in.*

4. Reduce 2 miles to chains. Reduce 5 miles to chains.

5. Reduce 2½ *sq. yds.* to *sq. ft.* Reduce 5 *sq. ft.* to *sq. in.*

6. How many square chains in 12 acres? In 15 acres?

7. How many quarts in 4 gallons and 1 quart?

8. In 5 bushels and 3 pecks how many pecks?

9. In 4 *lbs.* Avoirdupois how many ounces?

10. How many scruples in 6 *drs.*? In 12 *drs.*?

11. How many pecks in 3 bushels of wheat?

12. How many minutes in 6 hours and 10 minutes?

13. How many pints in 7 quarts? In 10 *qts.*?

14. How many pence in 3 crowns? In 8½ crowns?

15. How many degrees in 2 right angles? In 4 right angles?

EXAMPLES FOR WRITTEN WORK.

Reduce £18 14*s.* 9*d.* 3 *far.* to farthings.

ILLUSTRATION.

£18 14*s.* 9*d.* 3 *far.*
20
294
12
3537
4
14151 *far.*

EXPLANATION.—Since £1 is equal to 20 shillings, there are 20 times as many shillings as pounds, to which we add 14*s.* (18 × 20) + 14 = 294*s.* Since 1 shilling is equal to 12 pence, there are 12 times as many pence as shillings: to this we add 9 pence. (294 × 12) + 9 = 3537*d.* Since 1 penny is equal to 4 farthings, there are 4 times as many farthings as pence, to which add 3 farthings. (3537 × 4) + 3 = 14151 *far.* Hence the

RULE.

I. Multiply the units of the highest denomination by the number of the scale that connects this denomination with the one next lower, and to the product add the units of the latter denomination.

II. Multiply this result by the number that connects it with the next lower denomination, and to the product add the units of that denomination.

III. Continue this operation till the required denomination is reached.

EXAMPLES.

1. Reduce 3 *lbs.* 4 *oz.* 5 *dwts.* Troy, to pennyweights.

2. Reduce 1 *lb.* 7 *oz.* 15 *dwts.* 4 *grs.* Troy, to grains.

3. Reduce 7 ʒ 2 ℈ 15 *grs.* to grains.

4. Reduce 4 ℥ 5 ʒ 2 ℈ to grains.

5. Reduce 3 *qrs.* 15 *lbs.* 4 *oz.* avoirdupois, to ounces.

6. Reduce 6 *T.* 10 *cwt.* 3 *qrs.* 4 *lbs.* to pounds.

7. Reduce 7 *wks.* 2 *da.* 4 *hrs.* to hours.

8. Reduce 4 *da.* 16 *hrs.* 42 *min.* to minutes.

9. Reduce 2 *wks.* 5 *da.* 8 *hrs.* to seconds.

10. Reduce 16 *rds.* 4 *yds.* 2 *ft.* 9 *in.* to inches.

11. Reduce 1 *mi.* 240 *rds.* 2 *yds.* to feet.

12. How many inches in 7 *ch.* 74 *li.*?

13. Reduce 4 *mi.* 3 *ch.* to links.

14. Reduce 40 *sq. rds.* 8 *sq. yds.* to square feet.

15. How many acres in 56 *sq. mi.*?

16. Reduce 2 *A.* 74 *sq. li.* to square links.

17. Reduce 20 *cu. ft.* 168 *cu. in.* to cubic inches.

18. How many pints in 4 *bu.* 3 *pks.* 6 *qts.*?

19. Reduce 5 *bu.* 2 *pks.* 4 *qts.* 1 *pt.* to pints.

EXAMPLES IN REDUCTION ASCENDING.

FOR ORAL WORK.

218. *1.* How many *ft.* in 216 *in.*? How many *yds.*?

2. How many dollars in 6400 cents? In 9600 cents?

3. In 54 square feet, how many square yards?

4. In 450 square chains, how many acres?

5. How many cubic yards in 162 cubic feet?

6. In 200 cord feet, how many cords of wood?

7. Reduce 360 square chains to acres.

8. How many hands in 164 inches?

9. Reduce 224 pints to pecks.

10. Reduce 92 quarts to gallons.

11. In 96 ounces, how many pounds Troy?

12. Reduce 48℈ to ounces?

13. Reduce 480 *dwt.* to pounds.

14. Reduce 1200 minutes to hours.

15. How many degrees are 480 minutes?

16. Reduce 240° of longitude to hours.

17. How many minutes are 480 seconds?

18. How many dozen oranges are 480 oranges?

FOR WRITTEN WORK.

1. Reduce 8,743*d.* to pounds sterling.

ILLUSTRATION.

12) 8743
20) 728*s.* + 7*d.*
£36 + 8*s.*

EXPLANATION.—Since 12*d.* make 1*s.*, there are $\frac{1}{12}$ as many shillings as pence. Hence, 8,743*d.* = 728*s.*; the remainder 7 we express in pence. Since 20*s.* make £1, there are $\frac{1}{20}$ as many pounds as shillings = £36; the remainder 8 we express as shillings; hence, 8,743*d.* = £36 8*s.* 7*d.*

RULE.

I. Divide the given number by the number of the scale that connects it with the next higher denomination; the remainder will be units of the same denomination as the dividend.

II. Divide the quotient by the number that connects it with the next higher denomination; the remainder will be units of the denomination of the new dividend.

III. Continue the operation till the required denomination is reached.

2. Reduce 11,911 grains to pounds Troy.
Ans. 2 *lbs.* 16 *dwts.* 7 *grs.*

3. Reduce 876 *dwts.* to pounds Troy.

4. Reduce 1,511 *grs.* to ounces, apothecaries' weight.

5. Reduce 1,594 *oz.* avoirdupois, to quarters.

6. Reduce 8,842 *lbs.* to tons. Reduce 4,412 *lbs.* to *tons.*

7. Reduce 7,620 minutes to days.

8. Reduce 687 *hrs.* to weeks. Reduce 348 *hrs.* to *wks.*

9. How many yards in 786 *in.*? In 1572 *in.*?

10. How many miles in 897 *rds.*? In 1794 *rds.*

11. Reduce 2,875 *in.* to yards. Reduce 575 *in.* to *yds.*

12. Reduce 510 *pts.* to bushels. Reduce 255 *pts.* to *bu.*

13. Reduce 352 *qts.* to bushels. Reduce 704 *qts.* to *bu.*

14. How many gallons in 840 *pts.*? In 420 *pts.*

15. Reduce 8,532 *gi.* to gallons. Reduce 2,133 *gi.* to *gal.*

16. How many degrees in 8,844″? In 4,422′.

17. How many cubic yards in 162 cubic feet?

18. In 200 cord feet how many cords of wood?

19. Reduce 360 square chains to acres.

ADDITION OF COMPOUND NUMBERS.

219. Addition of Compound Numbers is the operation of finding the sum of two or more compound numbers.

The principles are the same as in simple numbers.

Exercises for Oral Work.

1. What is the sum of 9 inches and 11 inches? Of 22 *in.* and 13 *in.*? Of 8 *ft.* and 17 *ft.*? Of 9*d.* and 16*d.*

2. What is the sum of 2 *ft.* 4 *in.*, and 6 *ft.* 3 *in.*?

3. What is the sum of 2 *yds.* 1 *ft.*, and 3 *yds.* 4 *ft.*?

4. What is the sum of 2 *gals.* 3 *qts.*, and 3 *gals.* 3 *qts.*?

5. What is the sum of 3*s.* 8*d.*, and 5*s.* 9*d.*?

6. What is the sum of 3 *yds.* 3 *ft.*, 5 *yds.* 1 *ft.*, and 7 *yds.*?

7. What is the sum of 2 *wks.* 3 *das.*, and 3 *wks.* 4 *das.*?

8. What is the sum of 8 *sq. ft.* 16 *sq. in.*, and 5 *sq. ft.*? 4 *sq. in.*?

9. Find the sum of 3 *bush.* 1 *pk.*, and 4 *bush.* 2 *pks.*

10. What is the sum of $2^\circ\,3'\,4''$, and $3^\circ\,1'\,2''$, and $5^\circ\,2'\,3''$?

Operation of Addition of Compound Numbers.

Let it be required to find the sum of £7 4*s.* 3*d.*, £11 9*s.* 8*d.* and £14 12*s.* 9*d.*

ILLUSTRATION.

£	*s.*	*d.*
7	4	3
11	9	8
14	12	9
£33	6*s.*	8*d.*

EXPLANATION.—We write the numbers so that units of the same denomination shall stand in the same column. We add the right hand column, and find its sum is 20*d.* = 1*s.* 8*d.*; we write 8*d.* under the column in the place of the amount, and add 1*s.* with the column of shillings, which amounts to 26*s.* = £1 6*s.*; we write 6*s.* under the column, and add £1 with the column of £s., which amounts to £33. The required sum is therefore £33 6*s.* 8*d.* Hence, the following

RULE.

I. Write the numbers so that units of the same denomination shall stand in the same column.

II. Add the units of the lowest denomination, and divide their sum by the number of the scale that connects this denomination with the next higher one; set down the remainder, and carry the quotient to the next column.

III. Add the units of the second column thus increased, and proceed as before, continuing the operation till all the columns have been added.

EXAMPLES.

Perform the following indicated additions:

(*1.*)

£	*s.*	*d.*
1	14	8
3	12	7
5	2	9

(*2.*)

ʒ	℈	*grs.*
1	2	10
2	1	9
3	1	7

(*3.*)

cwt.	*qrs.*	*lbs.*
4	1	18
2	3	7
11	2	13

(*4.*)

wks.	*da.*	*hrs.*
2	5	7
3	4	11
5	6	14
1	4	20

(*5.*)

yds.	*ft.*	*in.*
2	2	8
1	1	7
3	1	6
4	0	11

(*6.*)

°	′	″
20	15	20
18	13	15
21	14	30
16	30	10

(7.)			*(8.)*			*(9.)*			*(10.)*		
T.	*cwt.*	*qrs.*	*oz.*	*dwts.*	*grs.*	*gals.*	*qts.*	*pts.*	*bu.*	*pks.*	*qts.*
2	1	15	11	19	12	4	3	1	5	3	4
1	3	20	4	16	5	3	2	0	7	1	5
3	2	18	2	11	15	7	1	0	11	0	3
5	3	7	6	3	7	6	2	1	3	2	6
1	2	23	1	15	4	2	0	1	6	3	1

What is addition of compound numbers? What compound numbers can be added? Give the rule for addition of compound numbers.

SUBTRACTION OF COMPOUND NUMBERS.

220. Subtraction of Compound Numbers is the operation of finding the difference between two compound numbers.

The principles are the same as in subtraction of simple numbers.

Exercises for Oral Work.

1. From 11 *in.* subtract 6 *in.* From 12 *ft.* subtract 9 *ft.*

2. What is the difference of 7 *pks.* and 4 *pks.*?

3. What is the difference between £3 4*s.*, and £1 2*s.*?

4. What is the difference between 2*s.* 7*d.*, and 1*s.* 4*d.*?

5. 8 *yds.* 2 *ft.* — 6 *yds.* 1 *ft.* = ?

6. 7 *gals.* 1 *qt.* — 5 *gals.* 3 *qts.* = ?

7. 15 *ft.* 8 *in.* — 9 *ft.* 11 *in.* = ?

Operations of Subtraction of Compound Numbers.

Let it be required to find the difference between £9 4*s.* 3*d.* and £2 18*s.* 6*d.*

ILLUSTRATION.

£	s.	d.
9	4	3
2	18	6
£6	5*s.*	9*d.*

EXPLANATION.—We write the subtrahend under the minuend, so that units of the same denomination shall stand in the same column. We begin at the right. Since 6*d.* is greater than 3*d.* we add 12*d.* to 3*d.* making 15*d.*, from which we subtract 6*d.*, and write the remainder 9*d.* beneath. To compensate for 12*d.* added to the minuend, we add 1*s.* equal to 12*d.* to the next number of the subtrahend, obtaining 19*s.*, which being greater than 4*s.*, we add 20*s.* to 4*s.*, obtaining 24*s.*, from which we subtract 19*s.*, and write the remainder 5*s.* beneath. To compensate for 20*s.* added to the minuend, we add £1 equal to 20*s.* to the subtrahend, obtaining £3, which we subtract from £9, and write the remainder £6 beneath; hence, the remainder is £6 5*s.* 9*d.*

In like manner we treat all similar cases; hence, the following

RULE.

I. Write the subtrahend under the minuend so that units of the same denomination shall stand in the same column.

II. Subtract each number in the subtrahend from the one above it in the minuend, and write the remainder in the line below.

III. If any number in the subtrahend is greater than the one above it in the minuend, add to the latter as many units as make one of the next higher denomination, perform the subtraction, and add 1 to the next number in the subtrahend.

EXAMPLES.

	(1.)			*(2.)*			*(3.)*			*(4.)*		
	£	s.	d.	da.	hrs.	min,	yds.	ft.	in.	oz.	wts.	grs.
From	9	5	2	57	21	43	46	1	7	13	18	5
Subtract	6	3	1	49	23	39	35	2	10	9	19	23
Remainder,	3	2	1	7	22	4	10	1	9	3	18	6

(5.)

	tun.	hhd.	gal.	qt.	pt.
From	151	3	50	3	2
Take	27	2	54	3	2

(6.)

	yr.	wk.	da.	hr.	min.	sec.
From	95	25	4	20	45	50
Take	80	30	6	23	46	56

INTERVAL BETWEEN DATES.

To find the interval between two dates, we subtract the earlier from the later. To do this, we write the number of the year, the number of the month, and the day of the month of each date; we then perform the subtraction, counting 30 days to a month, and 12 months to a year.

ILLUSTRATION.

Dates.	*yrs.*	*mos.*	*da.*
June 3, 1776, written . .	1776	6	3
Oct. 16, 1771, " . .	1771	10	16
Interval . . .	4 *yrs.*	7 *mos.*	17 *da.*

EXPLANATION.—Having written the dates as shown on the right, we proceed according to the preceding rule.

8. What is the interval between Nov. 10, 1862, and March 4, 1875?

9. What is the interval between Sept. 21, 1851, and Feb. 11, 1873?

MULTIPLICATION OF COMPOUND NUMBERS.

221. Multiplication of Compound Numbers is the operation of taking a compound number as many times as there are units in the multiplier.

The principles are the same as in the multiplication of simple numbers.

EXERCISES FOR ORAL WORK.

1. How many inches are 7 times 8 inches?

2. What is the product of 8*d.* multiplied by 11?

Express the answer in shillings and pence; thus

8*d.* × 11 = 88*d.* = 4*s.* 8*d.* *Ans.*

3. What is the product of 2 *ft.* multiplied by 16?

Express the answer in *yds.* and *ft.*

4. What is the product of 7 *qts.* multiplied by 13?

5. What is the product of 1 *ft.* 3 *in.* multiplied by 9?

6. What is the product of 4 *bu.* 3 *pks.* multiplied by 9?

7. What is the product of 1*s.* 7*d.* multiplied by 8?

8. What is the product of 8 *gals.* 2 *qts.* multiplied by 4?

9. Find the product of 2 *tons* 3 *cwt.* multiplied by 6?

10. What is the product of 2° 3′ 1″ multiplied by 5?

OPERATION OF MULTIPLICATION OF COMPOUND NUMBERS.

Let it be required to multiply £4 2*s.* 5*d.* by 16.

ILLUSTRATION.

£	*s.*	*d.*
4	2	5
		16
£65	18*s.*	8*d.*

EXPLANATION.—We write the multiplier under the multiplicand, then multiply 5*d.* by 16, obtaining 80*d.* = 6*s.* 8*d.*; we write 8*d.* beneath, and add 6*s.* to the product of 2*s.* × 16, obtaining 38*s.* = £1 18*s.*; we write 18*s.* beneath and add £1 to the product £4 × 16, obtaining £65, which we write beneath; hence the following

RULE.

I. Multiply the units of the lowest denomination in the multiplicand by the multiplier, and divide the product by the number of the scale that connects this denomination with the one next higher; set down the remainder.

II. Multiply the units of the next higher denomination in the multiplicand by the multiplier, add the quotient to the product, and divide the sum by the number of the scale that connects this denomination with the one next higher: set down the remainder and preserve the quotient to be added to the next product.

III. Continue this operation till all the parts of the multiplicand have been multiplied.

EXAMPLES.

Perform the following multiplications:

(*1.*)

bu.	*pks.*	*qts.*
3	2	7
		11
40	3	5

(*2.*)

gals.	*qts.*	*pts.*
5	2	1
		12
67	2	0

(*3.*)

£	*s.*	*d.*
5	4	9
		7
36	13	3

(*4.*)

T.	*cwt.*	*lbs.*	*oz.*
2	14	10	12
			15
40	16	11	4

(*5.*)

wk.	*da.*	*hrs.*	*min.*	*sec.*
2	4	7	15	18
				12
31	2	15	3	36

6. Multiply 17 *lbs.* 5 *oz.* 12 *dwts.* 16 *grs.* Troy, by 9.

7. Multiply 4 *cwt.* 3 *qrs.* 8 *lbs.* by 28.

8. Multiply 17 *da.* 14 *hrs.* 14 *min.* 15 *sec.* by 12.

9. Multiply 16° 11′ 13″ by 7.

DIVISION OF COMPOUND NUMBERS.

223. Division of Compound Numbers is the operation of dividing a compound number by an abstract number, or by a similar denominate number.

The principles are the same as those used in the division of simple numbers.

EXERCISES FOR ORAL WORK.

1. Divide 35 nuts equally among 7 boys.

2. What is the quotient of 18*s.* divided by 6? By 9?

3. What is the quotient of 48 *yds.* divided by 12?

4. Divide 12*s.* 9*d.* by 3. Divide 18*s.* 6*d.* by 6.

5. What is the quotient of 84 *yds.* 3 *quarters* divided by 3?

6. Divide 57 *lbs.* by 19 *lbs.* Divide 114 *lbs.* by 19 *lbs.*

7. What is the quotient of 84 *yds.* divided by 12 *yds.*?

8. Divide 9 *yds.* 2 *ft.* 6 *in.* by 3. Divide 18 *yds.*

9. Divide 24° 30′ 36″ by 6. Divide 48° 24′ by 8.

10. A floor contains 40 *sq. yds.*, and its length is 8 *sq. yds.*; what is its breadth?

OPERATION OF DIVISION OF COMPOUND NUMBERS.

Let it be required to divide £65 18*s.* 8*d.* by 16.

ILLUSTRATION.

```
      £   s.  d.
16 ) 65  18   8 ( £4 2s. 5d.
     64
     £1
      20
      38s.
      32
       6s.
      12
      80d.
      80
```

EXPLANATION.—Dividing £65 by 16 gives a quotient of £4 and a remainder £1. Reducing £1 to shillings and adding 18*s.*, we have 38*s.*; dividing 38*s.* by 16, gives a quotient 2*s.* and a remainder 6*s.* Reducing 6*s.* to pence and adding 8*d.*, we have 80*d.*; dividing 80*d.* by 16, we find a quotient 5*d.* Hence the following

RULE

I. Divide the units of the highest denomination in the dividend by the divisor; set down the quotient as a part of the required quotient.

II. Reduce the remainder to the next lower denomination, and to the result add the units of that denomination for a new dividend, and proceed as before.

III. Continue this operation till the division is completed.

EXAMPLES.

1. Divide 24 *da.* 19 *hrs.* 30 *min.* by 6.

2. Divide 43 *wks.* 2 *da.* 9 *hrs.* by 9.

3. Divide 39 *bu.* 2 *pks.* 1 *qt.* by 11.

4. Divide 1 *T.* 19 *cwt.* 2 *qrs.* 12 *lbs.* by 7.

5. Divide 56 *lbs.* 6 *oz.* 17 *dwts.* by 9.

6. Divids 14 *cwt.* 1 *qr.* 8 *lbs.* by 13.

7. Divide 13 *bu.* 3 *pks.* by 2 *bu.* 3 *pks.*

Explanation.—The dividend is equal to 55 *pks.*, and the divisor is equal to 11 *pks.*; hence the quotient of 13 *bu.* 3 *pks.* ÷ 2 *bu.* 3 *pks.* is equal to 55 *pks.* ÷ 11 *pks.* = 12, *Ans.*

All similar cases may be worked by the following

RULE.

Reduce both numbers to the same denomination and divide as in simple numbers.

8. Divide 21 *wks.* 5 *da.* by 2 *wks.* 5 *da.*

9. Divide 2 *qrs.* 12 *lbs.* 9 *oz.* by 5 *lbs.* 11 *oz.*

10. Divide 19 *bu.* 3 *pks.* 3 *qts.* by 3 *bu.* 3 *pks.* 7 *qts.*

REVIEW QUESTIONS.

What is a bill? An account? A debtor? A creditor? How is a bill receipted? What is a footing? What is debit? Write the abbreviations of at, account, amount, balance, company, creditor, debtor, paid, and received. How do you find the footing of a bill? How the balance? What is reduction? Define reduction descending. Define reduction ascending. Recite the table of troy weight. What is its primary unit? Recite the table of avoirdupois weight. Recite the table of long measure. What is its unit of measure? What is magnitude? What is a line? A curved line? A straight line? What are parallel lines? Recite the table of surveyor's measure. For what is it used? What is a surface? An angle? Illustrate an angle, and name its parts. Define a square. A square foot. A square yard. What is the primary unit of the measure of surface? What is an area? How is the area of a square or rectangle found? Recite the table of square measure. Recite the table of fluid measure. For what used? What is its primary unit? Recite the table of dry measure. For what used? What is the primary unit? Describe the bushel. What is a circle? What the circumference? Diameter? Radius? Recite the table of angular measure. What is its primary unit? What is longitude? Recite the table of longitude. What is a volume? A cube? Faces of a cube? Edges? Recite the table of cubic measure. What is a cubic foot? A cord foot? A cord of wood? How do you reduce pounds sterling to shillings? How farthings to pounds? What fundamental rule is used in reduction descending? In reduction ascending? Tell the number of days in each month of the civil year. How can you tell when a year is a common year? How, when it is leap year? How many days in each? Is 1878 a leap year? Will the year 2000 be common, or leap year? How many days were there in the year 1800? When will the next leap year occur? What is compound addition? What compound numbers can be added? Give the rule for addition of compound numbers? What is subtraction of compound numbers? Give the rule. How do you find the interval between two dates? What is multiplication of compound numbers? Give the rule. What is division of compound numbers? How many cases are there? Give the rule when the divisor is abstract. Give the rule when the divisor is similar to the dividend.

NOTE.—For a more extended treatment of Denominate Numbers, see Davies & Peck's Complete Arithmetic.

PERCENTAGE AND ITS APPLICATIONS.

224. Per cent means by the hundred, or hundredths. Thus, 3 per cent of $100 is $\frac{3}{100}$ of $100, or $3.

1. What is 2 per cent of 100? Of 300? Of 60?

2. How many dollars is 5 per cent of $100? Of $50?

3. How many *yds.* is 7 per cent of 100 *yds*? Of 500 *yds.*?

225. The sign of per cent is %. Thus, 3% of 20 is read 3 per cent of 20.

Read the following examples:

4. 4% of 20; 5% of 100; 6% of 30; 7% of $200.

5. 1% of $100; 9% of 27; 10% of 33 feet; 11% of 50.

6. 12% of 17 bushels; 20% of 450 ships; 30% of 72.

226. The rate per cent, or simply **rate,** is the number of hundredths taken; thus, in the expression 7% of 245, the rate is 7 *hundredths.*

1. How many per cent is .04? .06? .08? .07? .12? .17?

2. How many hundredths is 6%? 3%? 5%? 7%? 11%?

3. How many hundredths is 18%? 17%? 21%? 25%?

4. How many per cent is .25? .31? .15? .18? .09?

5. What is the decimal expression for $1\frac{1}{2}\%$?

SOLUTION. $.01\frac{1}{2} = .015$.

Per cent expressed by means of a common fraction may be expressed decimally by annexing two ciphers to the denominator, and reducing the result to a decimal.

ILLUSTRATION. $\frac{4}{5}\% = \frac{4}{500} = .008$.

6. What decimal fraction is $\frac{1}{2}\%$?

7. Express $\frac{3}{4}\%$ decimally. *8.* Express $\frac{2}{5}\%$ decimally.

9. Express $\frac{1}{5}\%$ decimally. *10.* What decimal fraction is $\frac{3}{5}\%$?

11. How many hundredths is 100%? 200%?

12. How many hundredths is 125%? Express it decimally.

SOLUTION. 125% = $\frac{125}{100}$ = 1.25, *Ans.*

13. Express 250% decimally.

227. Percentage is some per cent of a given number. Thus, $6 is the percentage on $100, when the rate is 6 per cent.

1. What is the percentage on $200 at 5 per cent?

2. What is the percentage on $3528 at 2 per cent?

3. What is the percentage on $230 at 7 per cent?

4. What is the percentage on 350 yards, at rate of 3%?

5. Find the percentage on $15 at 50 per cent.

228. Base is the number on which percentage is reckoned. In the expression 7% of $100, the base is $100.

229. Amount is the base increased by the percentage. The amount of $100 at 8 per cent, is $100 + $8 = $108.

1. What is the amount of $200 at 3 per cent?

2. If 500 is the base, and 6 the rate, what is the amount? What, if 600 is the base, and 7 the rate?

3. If $648 is the base, and $16 the percentage, what is the amount?

4. Find the amount when the base is $4000, at 2%.

5. Find the amount when the rate is 6, and the base is $456.

230. The Difference is the base diminished by the percentage. Thus, the difference of $100 diminished by *8 per cent, is* $100 — $8 = $92.

1. What is $600 diminished by 3 per cent?

2. Find the difference when the base is $875 and the percentage $26.25.

3. Find the difference when the base is $10,000 and the rate is 6%.

4. What is the difference when the base is $275 and the rate is 4%?

5. What is the difference when the base is $72 and the rate is 30%?

EXERCISES FOR ORAL WORK.

1. What is 5 per cent of 40 *lbs.*?

2. If $80 is increased by 5% of itself, what is the amount?

3. What is the difference of 80 yards diminished by 3 per cent of itself?

4. What per cent of $100 is $7? 8? 18? 20?

5. The base is $200 and the rate 7%; what is the amount? What is the difference?

6. A man had 30 chickens, 20 per cent of them were destroyed by foxes; how many were destroyed? How many were left?

7. How many marbles are 7% of 500 marbles?

8. A boy answered 25 questions in arithmetic, his brother answered 60% of that number; how many questions did the brother answer?

9. A man who held $10,000 worth of United States bonds, sold 10% of them; what value of bonds did he sell? What value of bonds had he left?

10. Let the base be $20,000 and the rate 6%; what is the *percentage*? What the amount? What the difference?

PRINCIPLES.

231. From what precedes we have the following principles:

1. *The percentage is equal to the base multiplied by the rate expressed decimally.*

2. *The amount is equal to the base multiplied by 1 plus the rate expressed decimally.*

3. *The difference is equal to the base multiplied by 1 minus the rate expressed decimally.*

Since either of two factors is equal to their product divided by the other, we have **the following principles:**

4. *The rate is equal to the percentage, divided by the base.*

5. *The base is equal to the percentage divided by the rate expressed decimally; to the amount divided by 1 plus the rate; or to the difference divided by 1 minus the rate expressed decimally.*

232. *To find the Percentage, when the base and rate are given.*

See Principle 1, Art. 231.

EXAMPLES.

1. What is 5% of 75 *lbs.*?

Solution. 75 *lbs.* × .05 = *Ans.* 3.75 *lbs.*

2. What is 7% of 115 *lbs.*? Of 25 *lbs.*? Of 50 *lbs.*?

3. What is 11% of $315? Of $248? Of $600?

4. What is 16% of 52 *wks.*? Of 20 *wks.*? Of 50 *wks.*?

5. What is 25% of 4,120 *yds.*? Of 5640 *yds.*?

6. What is 40% of 72 *bu.*? Of 300 *bu.*?

7. What is 120% of $250? Of $840? Of $784?

8. What is 100% of 87 *ft.*? Of 500 *ft.*? Of 600 *ft.*?

9. What is 210% of 40 *gals.*? Of 63 *gals.*? Of 94 *gals.*?

10. What is 60% of $60? Of $500?

11. What is ⅜% of $1,000? Of $8,000? Of $6,000?

233. *To find the Amount, when the Base and Rate are given.*

See Principle 2, Art. 231.

EXAMPLES.

1. What is the amount of 150 *lbs.* increased by 10% of itself? 150 *lbs.* × 1.10 = *Ans.* 165 *lbs.*

2. What is the amount of $300 increased by 35%?

3. What is the amount of 610 *yds.* increased by 16%?

4. What is the amount of 76 acres increased by 15%?

5. What is the amount of $218 increased by 9%?

6. What is the amount of 48 tons increased by 80%?

234. *To find the Difference, when the Base and Rate are given.*

See Principle 3, Art. 231.

EXAMPLES.

1. What is the difference between $108 and 30% of itself? $108 × .70 = $75.60, *Ans.*

2. What is the difference between 160 *rods* and 80% of itself?

3. Diminish $540 by 30% of itself.

4. Diminish 64 weeks by 8% of itself.

5. Diminish 880 *yds.* by 30% of itself.

6. Diminish $1,050 by 7% of itself.

235. *To find the Rate, when the Base and Percentage are given.*

See Principle 4, Art. 231.

EXAMPLES.

1. The percentage is $90.24, and the base is $752; what is the rate?

$90.24 ÷ 752 = .12 = 12 per cent, *Ans.*

2. What is the rate, when the percentage is $7 and the base is $100? What, when the base is $500?

3. Find the rate, when the base is $400 and the percentage is $20.

4. What is the rate, when the percentage is 60 *lbs.* and the base is 300 *lbs.*?

5. Given the base $2,000, and the percentage $200; what is the rate?

6. A gentleman in Cleveland sends to his friend in New York $50,000, asking him to take out $500 for his services, and invest the balance in New York Central R. R. stock; what rate per cent of the money sent does he pay for the services of his friend? What rate per cent of the money invested does he pay?

236. *To find the Base, when the Rate and Percentage are given.*

See Principle 5, Art. 231.

EXAMPLES.

1. 960 is 25 per cent of what number?

SOLUTION. 960 ÷ .25 = *Ans.* 3,840.

2. 74 is 62½ per cent of what number?

3. 450 is 112 per cent of what number?

4. Of what number is 66, 2 per cent?

5. In a school 100 pupils are present; this number is 80 per cent of the pupils on the roll; what is the number on the roll?

MISCELLANEOUS PROBLEMS.

1. The population of a town in 1860 was 3,750, and in 10 years it increased 30%; what was the population in 1870?

2. A merchant bought goods to the amount of $3,150 and paid 15% on their first cost for transportation and insurance; what was their final cost?

3. A man has 50 geese, 40% more of chickens than geese, and 60% more of ducks than chickens; how many chickens has he, and how many ducks?

4. A cask of wine contained 44 *gals.*, but 18% of it leaked out; how much remained?

5. A man has a capital of $20,000, of which he loses 35%; how much has he remaining?

Define per cent. What is percentage? What is the base? What is the rate? Illustrate. What is the amount? What the difference? Repeat the 5 principles of percentage. Write an example that will illustrate each principle.

COMMISSION.

237. Commission is a percentage paid to an agent for transacting business.

238. An **Agent** is one who transacts business for another. If he buys and sells merchandise, he is called a **Commission Merchant, or Factor**; if he buys and sells stocks, exchange, real estate and the like, he is called a **Broker**; if he collects debts, taxes, and the like, he is called a **Collector**.

239. A Consignment is a quantity of merchandise sent to an agent for sale. The party that sends the goods is called the **Consignor**; and the party to whom they are sent is called the **Consignee.**

240. An Account of Sales is an account rendered by the Consignee to the Consignor.

241. The Net Proceeds is the remainder after deducting commission and other expenses.

All problems in Commission are solved by the principles of percentage.

The base in Commission is generally what the agent expends or collects; but in buying and selling stocks, and the like, the commission is based on the par value.

EXAMPLES.

1. A commission merchant received a consignment of wheat which he sold for $2,850; what was his commission at the rate of 4% ? *Ans.* $114.

2. A commission merchant purchased cotton for a manufacturer to the amount of $5,140; what was his commission at 2% ? *Ans.* $102.80.

3. A broker sells a house for $13,400; what is his commission at 1⅛% ? *Ans.* $150.75.

4. A broker buys 6 lots of land for $8,490; what is his commission at ⅞% ?

5. A cotton broker sells 40 bales of cotton, each weighing 450 *lbs.*, for 15 *cts.* per pound; what is his commission at 1¾% ?

6. My agent purchased goods for me to the amount of $7,850, on which I agreed to pay 2¼% commission; how much must I pay him ?

7. An architect builds a house which costs $13,710, and charges 2½% for his services; what is his bill ?

8. A broker sold 70 shares of stock whose par value was $100 per share; what was his commission at ¼% ?

9. What commission should a broker receive for selling 100 shares of stock whose par value is $100, at ¼% ?

10. Find the net proceeds of the following account of sales:

Sales on account of JOHN R. BENSON, Rochester, N. Y.

1878.	DESCRIPTION.	$	CTS.
Jan. 4	200 *bbls.* flour @ $8 - - -		
17	115 *bu.* wheat @ $1.10 -		
20	1000 *lbs.* butter @ 30*c.* - -		
	Gross amount		
	CHARGES.		
	Freight and cartage, $48.75		
	Storage - - - - 10.50		
	Commiss'n on $ @ 4% ____		
	Net proceeds - - - $		

What is commission? What is an agent? A commission merchant? A broker? A collector? A consignment? A consignor? A consignee? What is the net proceeds? How do you solve problems in commission? What is the base? What is account of sales?

NOTE.—For a more extended treatment of Commission, see Davies & Peck's Complete Arithmetic.

PROFIT AND LOSS.

242. Profit and Loss are commercial terms indicating *gain* or *loss* in business transactions.

If the selling price of any article is greater than the cost price, there is a **profit**; if the selling price is less than the cost price, there is a **loss.**

The cost price is usually taken as the base.

Problems in profit and loss are solved by the principles of percentage.

EXAMPLES.

1. A man bought a horse for $220 and sold him at an advance of 15% ; what did he gain?

The gain is the percentage.

The base is $220; hence, $220 × .15 = $33, the gain.

2. A merchant bought goods to the amount of $1,200, and sold them at a loss of 8%; how much did he lose?

3. A grocer bought sugar for $650, and sold it at an advance of 20%; what did he receive for it?

4. A dealer bought tea for $940, but was obliged to sell it at a loss of 12%; what did he get for it?

5. For what must a house, that cost $11,000, be sold so as to gain 15% on the cost price?

6. Coffee was bought at 22 *cts.* a pound; for what must it be sold to gain 20%?

7. A merchant bought 100 pieces of muslin at $7 a piece, and sold it at a gain of 16½%; what did he receive for the lot?

What is profit and loss? When is there a profit in business? When is there a loss in business? What is the base? By what principles do we solve problems in profit and loss?

NOTE.—For the application of percentage to Insurance. Taxes, etc., see Davies & Peck's Complete Arithmetic.

SIMPLE INTEREST.

243. Interest is a percentage paid for the use of money. It is reckoned at a certain rate per cent for each year.

The **Principal** is the sum on which interest is computed; the **Rate** is the per cent for 1 year; and the **Amount** is the sum of the principal and interest for any given time.

244. Legal Rate is the rate of interest fixed by law.

Interest depends on the *principal,* the *rate,* and the *time.*

EXERCISES FOR ORAL WORK.

1. What is the interest on $25 for 1 year, at 6 per cent?

$25 × .06 = *Ans.* $1.50.

2. What is the interest on $15 for 1 year, at 7%?

What is the *principal* in each of these examples?

What is the rate? What the time?

3. What is the interest on $30 at 7%, for two years?

$30 × .07 = $2.10 = int. for 1 year; the int. for 2 years is twice the int. for 1 year; $2.10 × 2 = $4.20, *Ans.*

4. What is the interest on $100 for 4 years, at 7%?

5. What is the interest on $200 for 3 years, at 6%?

6. What is the interest on $500 for 5 years, at 7%?

7. What is the amount of $300 for 1 year, at 7%?

The amount is the sum of the principal and interest?

$300 × .07 = $21, the int.; $21 + $300 = *Am't.* $321.

8. Find the amount of $200 for 2 years, at $6%.

9. What is the amount of $300 for 3 years, at 7%?
10. Find the amount of $400 for 2 years, at 7%?

245. ***When the time is given in years, the interest may be found by the following***

RULE.

Multiply the principal by the rate, and multiply that result by the number of years.

EXAMPLES.

1. What is the interest on $85 for 4 years, at 7%?
Ans. $85 × .07 × 4 = $23.80.
2. What is the interest on $96 for 3 years, at 6%?
Ans. $96 × .06 × 3 = $17.28.
3. What is the amount of $336 for 5 years, at 5%?
4. What is the amount of $242 for 3 years, at 7%?
5. What is the interest on $425 for 5 years, at 7%?
6. What is the interest on $650 for 2 years, at 7%?
7. What is the interest on $780 for 3 years, at 6%?
8. What is the interest on $850 for 5 years, at 6%?
9. What is the interest on $1,200 for 7 years, at 7%?
10. What is the interest on $1,400 for 3 years, at 7%?
11. What is the interest on $890 for 4 years, at 5%?
12. What is the amount of $510 for 3 years, at 7%?
13. What is the amount of $1,240 for 4 years, at 7%?
14. What is the amount of $1,780 for 3 years, at 6%?
15. What is the amount of $1,672.15 for 4 years, at 7%?
16. What is the amount of $1,587.73 for 3 years, at 6%?

246. *To find the interest when the time is any number of months.*

ALIQUOT PARTS OF A YEAR.

12 mo. are 1 year,
11 mo. are $\frac{11}{12}$ of 1 year.
10 mo. are $\frac{5}{6}$ of 1 year,
9 mo. are $\frac{3}{4}$ of 1 year,
8 mo. are $\frac{2}{3}$ of 1 year,
7 mo. are $\frac{7}{12}$ of 1 year,
6 mo. are $\frac{1}{2}$ of 1 year,
5 mo. are $\frac{5}{12}$ of 1 year,
4 mo. are $\frac{1}{3}$ of 1 year,
3 mo. are $\frac{1}{4}$ of 1 year,
2 mo. are $\frac{1}{6}$ of 1 year,
1 mo. is $\frac{1}{12}$ of 1 year.

EXERCISES FOR ORAL WORK.

1. What is the interest on $100 for 6 months, at 7%?

Since the interest on $100 at 7% for 1 year is $7, for 6 months, the half of 1 year, it is $\frac{1}{2}$ of $7, or $3.50. ***Ans.***

2. What is the interest on $200 for 3 months, at 7%?

Since the interest on $200 at 7% for 1 year is $14, for 3 months, the $\frac{1}{4}$ of 1 year, it is $\frac{1}{4}$ of $14, or $3.50. ***Ans.***

RULE.

I. Find the interest for one year; take such part of this interest as the given number of months is part of 12 months or 1 year; or,

II. Multiply the principal by the rate and divide the product by 12; then multiply the quotient by the number of months.

EXAMPLES FOR WRITTEN WORK.

3. What is the interest on $300 for 6 months, at 6%?

4. What is the interest on $400 for 4 months, at 7%?

5. What is the interest on $200 for 5 months, at 6%?

6. Find the interest on $240 for 4 months, at 8%.

7. Find the interest on $553 for 5 months, at 6%.

8. Find the amount of $150.25 for 6 months, at 8%.

9. What is the amount of $204 for 11 months, at 7%?

10. What is the interest on $228 for 9 months, at 6%?

11. Find the amount of $137.50 for 8 months, at 6%.

12. What is the amount of $7596 for 10 months, at 8%?

RULE WHEN THE RATE IS 6 PER CENT.

Multiply the principal by half the number of months, and move the decimal point two places to the left hand.

247. *To find the interest when the time is any number of days.*

For the sake of convenience in business transactions, 30 days are generally considered 1 month.

ALIQUOT PARTS OF A MONTH.

To find the aliquot parts of a month write the number of days for a numerator, and 30 for a denominator; and reduce the fraction to its lowest terms; Thus, 25 days are $\frac{25}{30} = \frac{5}{6}$ of a month; 15 days are $\frac{15}{30} = \frac{1}{2}$ of a month.

EXERCISES.

1. If the interest on a certain sum be $60 for 1 month, how many dollars will it be for 1 day?

Since 1 day is $\frac{1}{30}$ of 1 month, the interest for 1 day is $\frac{1}{30}$ of the *interest for 1* month; $\frac{1}{30}$ of $60 = $2, Ans.

2. The interest on $400 for 1 month, at 9%, is $3; what is the interest on $400, at 9%, for 15 days?

Since 15 days are the $\frac{1}{2}$ of 1 month, the interest for 15 days is $\frac{1}{2}$ of the interest for 1 month; $\frac{1}{2}$ of $3 is $1.50, ***Ans.***

3. If $6 is the interest of a certain sum of money for 1 month, what is the interest of the same sum for 20 days?

Since 20 days are $\frac{2}{3}$ of 1 month, the interest for 20 days is $\frac{2}{3}$ of $6, or $4, ***Ans.***

4. The interest on $900 at a certain rate for 1 month is $5.25; what is it for 6 days?

Since 6 days are $\frac{1}{5}$ of 1 month, the interest for 6 days is $\frac{1}{5}$ of $5.25 = $1.05, ***Ans.***

From these illustrations we deduce the following

RULE.

I. Find the interest for 1 month, take such a part of this interest as the given number of days is part of 30 days or 1 month; or,

II. Divide 1 month's interest by 30, and multiply the quotient by the given number of days.

EXAMPLES.

1. What is the interest on $460 for 15 days, at 7%?

2. What is the interest on $784, at 7%, for 20 days?

3. What is the interest on $1200 for 10 days, at 6%?

4. What is the interest on $184 for 21 days, at 7%?

5. What is the interest on $250 for 18 days, at 6%?

6. What is the interest on $375 for 25 days, at 7%?

7. What is the interest on $450 for 6 days, at 9%?

8. What is the interest on $500 for 7 days, at 7%?

248. *To find the interest when the time is in years, months, and days.*

Find the interest on $360 for 3 years 7 months and 25 days.

ILLUSTRATION.

		360		
		.07		
	2)	25.20	Int. for 1 *yr.*	
		3		
		75.60	Int. for 3 *yrs.*	
	6)	12.60	Int. for 6 *mo.*	7 *mo.*
3)	2)	2.10	Int. for 1 *mo.*	
		1.05	Int. for 15 *da.*	25 *da.*
		70	Int. for 10 *da.*	
		$92.05	Int. for 3 *yrs.* 7 *mo.* 25 *da.*	

EXPLANATION.—We find the int. for 1 year by multiplying the *principal* by the rate expressed decimally. We multiply the int. of 1 year by 3, to find the int. for 3 years. We then take ½ of the int. of 1 year for the int. for 6 *mo.*, and ⅙ of the interest for 6 months for the interest for 1 month. We then take ½ of the interest for 1 month for the interest for 15 days, and ⅓ of the interest for 1 month for the interest for 10 days; hence the interest for 3 *yrs.* 7 *mo.* 25 *da.* is $92.05.

SECOND ILLUSTRATION.

12)	25.20
	2.10
	7
	$14.70
30)	2.10
	.07
	25
	35
	14
	1.75

$360	
.07	
25.20	Int. for 1 year.
3	
75.60	Int. for 3 years.
14.70	Int. for 7 mos.
1.75	Int. for 25 days.
$92.05	Int. for 3 years, 7 months, and 25 days.

EXPLANATION.—We find the int. for 1 *yr.* and for 3 *yrs.*, as in the first illustration. We then divide 1 year's int. by 12, which gives us $2.10, the int. for 1 *mo.* Multiply the int. for 1 *mo.* by 7, *obtaining $14.70,* the int. for 7 mo. We then divide the int. for

1 *mo.* by 30 to find the int. for 1 *da.*, and multiply this int. by 25 to find the int. for 25 *da.*, which is $1.75. We then place the int. for 7 *mo.* and the int. for 25 *da.* under the int. for 3 years, and find their sum, $92.05.

RULE.

Find the interest for each period and add the results.

EXAMPLES FOR WRITTEN WORK.

1. What is the interest on $174 for 3 *yrs.* 2 *mos.*, at 6%? *Ans.* $33.06.

2. What is the interest on $350 for 2 *yrs.* 6 *mos.*, at 7%? *Ans.* $61.25.

3. What is the interest on $700 for 4 *yrs.* 7 *mos.*, at 7%?

4. What is the interest on $825 for 2 *yrs.* 9 *mos.*, at 6%?

5. What is the amount of $950 for 3 *yrs.* 6 *mos.*, at 7%?

6. What is the amount of $686 for 3 *yrs.* 8 *mos.*, at 6%?

7. What is the amt. of $1,218 for 1 *yr.* 11 *mos.*, at 7%?

8. What is the amt. of $1,250 for 2 *yrs.* 3 *mos.*, at 6%?

NOTES.

249. A **Promissory Note** is a written promise to pay a sum of money, either on demand, or at some particular time.

The party that signs the note is called the **Maker,** the party to whom it is payable is called the **Payee,** and the party that has legal possession of it is called the **Holder.**

A **Negotiable Note** is one that is payable either *to order*, or *to bearer*; the following is the

FORM OF A NEGOTIABLE NOTE.

$375.00. New York, July 2d, 1878.

For value received I promise to pay John Doe, or order, three hundred and seventy-five dollars on demand, with interest at 7 per cent.

Richard Roe.

John Doe is the *payee*. He can transfer it by writing his name across the back; he then becomes the *endorser*. Richard Roe is the *maker*, and the person to whom it is *transferred* becomes the *holder*.

The **Face** of the note is the sum named in it.

250. To find the interest on a note we have the following

RULE.

I. Subtract the date of the note from the date of settlement; the result will be the time.

II. Find the interest on the face of the note for this time.

EXAMPLES.

1. A note for $250 at 7%, bears date Oct. 10, 1877; how much interest will be due Dec. 13, 1878.

ILLUSTRATION.

yr.	*mo.*	*da.*
1878	12	13
1877	10	10
1 yr.	*2 mo.*	*3 da.*

EXPLANATION.—Here the time is 1 *yr.* 2 *mos.* 3 *da.*, and the interest on $250 for this time at the rate of 7%, is $20.562, *Ans.*

2. A note for $780 at 7%, bears date July 5, 1870; how much interest is due May 17, 1873? *Ans.* $156.52.

3. A note for $960 at 6% is dated Dec. 24, 1871; what amount is due on the note June 12, 1874?

4. A note for $1,140 at 7% is dated Jan. 11, 1872; what is its amount June 5, 1874?

PARTIAL PAYMENTS.

251. A **Partial Payment** is a payment of a *part* of the amount due on a note or other obligation.

The date and amount of each partial payment is *indorsed*, that is, *written on the back of the note,* and is to be taken into account at the settlement.

The following rule, for settling a note with endorsements, is adopted by the Supreme Court of the United States, and is called

THE SUPREME COURT RULE.

I. Find the amount of the given principal up to the time when the sum of the partial payments equals, or exceeds, the interest then due; from this subtract the sum of the partial payments up to the time considered.

II. Take the remainder for a new principal and proceed as before, continuing the operation to the time of settlement.

EXAMPLES.

1. On a note for $1,250 at 6%, dated June 10, 1876, are the following endorsements:

December 16, 1876,	$30.
July 4, 1877,	$250.

What is the amount due August 25, 1878?

OPERATION.

Principal, . . .	$1250
Int. to July 4, 1877, .	80
Am't July 4, 1877, .	$1330
Sum of payments, .	280
Remainder, ...	$1050
Int. to Aug. 25, 1878,	134.925
Amount, . .	$1184.925

EXPLANATION. — We see by inspection that the first payment is less than the interest then due; we therefore find the amount of the note up to the date of the second payment, which is $1,330; from this we subtract the sum of the first and second payments, and then find the amount of the remainder from the time of the second payment up to the date of settlement; this gives $1,184.925.

2. On a note for $960 at 7%, dated March 17, 1875, are the following endorsements:

March 17, 1876, $250.
March 17, 1877, $350.

What is due March 17, 1878? *Ans.* $515.3163.

DISCOUNT

252. Discount is a percentage deducted from the face of a bill, debt, or note.

COMMERCIAL DISCOUNT.

253. Commercial Discount is a percentage deducted from the face of a bill of merchandise.

The face of the bill is the **Base**, and the difference between this and the discount is called the **Net Proceeds**.

To find commercial discount we have the following

RULE.

I. Multiply the face of the bill by the rate per cent, and the product will be the discount.

II. Subtract the discount from the face of the bill, and the difference will be the net proceeds.

1. What is the discount on a bill of $350 at 2%? What the net proceeds?

2. Coal is sold on credit at $5 per ton; what is the cost price, the discount being 12%?

3. Bought a bill of goods whose face is $1,300, at a discount of 2½%; how much must I pay?

4. Sold 50 *bbls.* of flour at $7.50 per *bbl.*, deducting 7½% for cash; what was the net proceeds?

PRESENT VALUE AND TRUE DISCOUNT.

254. The **Present Value** of a debt, payable at a future time, is a sum which, being placed at interest, will give an amount equal to the debt when it falls due.

255. True Discount is the difference between the amount of the debt and its present value. Hence the

RULE.

I. Divide the amount of the debt by $1 plus the interest of $1 for the given time and at the given rate; the quotient will be the present value.

II. Subtract the present value from the amount of the debt; the remainder will be the true discount.

1. What is the present value of $500 due 2 years hence, money being worth 7%?

2. What is the true discount on a debt of $600 due 1 year hence, if paid now, at 6%?

BANKS AND BANK DISCOUNT.

256. **A Bank** is an incorporated institution, authorized by law to deal in money.

257. **Bank Discount** is a percentage charged for advancing money on a note or other obligation, payable at a future time.

258. A note is said to **Mature** when it becomes legally due, which is 3 days after it is nominally due. These 3 days are called **Days of Grace.**

FORM OF BANK NOTE.

$1000. New York, March 2, 1878.

Sixty days after date I promise to pay to the order of Alfred C. Barnes, one thousand dollars, at the Metropolitan Bank, New York.

Charles Martin.

Due May 1/4, 1878.

EXPLANATION.—Suppose this to be discounted on the day of its date. Alfred C. Barnes endorses it by writing his name across the back, and delivers it to the discount clerk. Interest is then *computed on the* face of the note for 63 days—the days of grace being added to the time mentioned in the note—at 7%, the legal rate in New York. This sum, $12.08, is the Bank discount. Subtracting the discount from $1000 we have $987.92 the proceeds. If the note is not paid before the close of banking hours of the last day of grace, May 4, a written notice, called a ***Protest***, is sent to Mr. Barnes, and he then becomes liable for its payment.

EXAMPLES.

1. A note for $2000, payable 60 days after date, is discounted at the rate of 7%; what is the proceeds?

2. A note for $3000, dated April 5, 1878, and payable 90 days after date, at 7%, is discounted on the day of its date; what is the proceeds?

REVIEW QUESTIONS.

What is interest? What is the principal? The rate? The amount? On what three elements does interest depend? Give the rule for interest when the time is given in years. When the time is given in months. Give the rule for interest when the time is given in days. Give the rule for interest when the time is given in years, months, and days. What is a promissory note? Who is the maker? The payee? The holder? When does the payee become an endorser? What is the rule for finding the interest on a note? What is a partial payment? How endorsed? Give the Supreme Court rule for settling a note with endorsements. What is discount? What is commercial discount? Give the rule to find commercial discount. What is present value? What is true discount? Give the rule to find true discount. What is a Bank? What is Bank discount? When is a note said to mature? What are days of grace? Write the form of a Bank note. Explain the method of discounting a Bank note.

NOTE.—For a full treatment of percentage and its applications, see Davies & Peck's Complete Arithmetic. See the same work for a full treatment of Proportion, Analysis, Square and Cube Roots, Mensuration, etc.

MISCELLANEOUS EXAMPLES.

1. A wagon wheel that turns round 346 times in running 1 mile, turns round 32,870 times in running from New York to Philadelphia: what is the distance between the two cities? *Ans.* 95 miles.

2. A gentleman bought 4 loads of hay; the first load weighed $1\frac{1}{4}$ tons, the second weighed 1 ton, the third weighed $\frac{1}{2}$ as much as the first and second together, and the fourth weighed $11\frac{1}{2}$ tons: how much hay did he buy? . *Ans.* $4\frac{7}{8}$ tons.

3. If $54 dollars will purchase 9 barrels of flour, how many barrels will $186 buy? *Ans.* 31 barrels.

4. A ship is 97 *ft.* 6 *in.* long; how many times her own length does she sail in running 78 miles? *Ans.* 4,224 times.

5. What is the bank discount on a note for $400, payable 90 days after date, at 6%? *Ans.* $6.20.

6. A. owned $\frac{1}{3}$ of a ship, and sold $\frac{4}{5}$ of his share for $13,500; what would the entire ship bring at the same rate? *Ans.* $80,625.

7. A. can do $\frac{2}{3}$ as much work as B., and B. can do $\frac{4}{5}$ as much as C.; if C. can do a piece of work in 10 days, how long will it take A. to do it? *Ans.* $18\frac{3}{4}$ days.

8. The longitude of Nantucket is 70° 5′ 39″, and the longitude of New York City is 74° 0′ 3″; what is the difference of longitude of the two places? *Ans.* 3° 54′ 24″.

9. A commission merchant received a consignment of corn, which he sold for $894, receiving a commission of $3\frac{1}{2}$%; what was his commission? *Ans.* $31.29.

10. A. and B. commenced business on the 1st of January with a capital of $5,000 each; during the year A. lost 40% of his capital, and B. gained 30% of his capital; how much more capital had B., at the end of the year, than A.? *Ans.* $3,500.

11. A. borrows $580 at a bank, payable in 60 days; what is the net proceeds, interest being reckoned at 7%? *Ans.* $651.05.

12. What is the cost of $4,000 of U. S. 5-20's at 110%, and brokerage at $\frac{1}{4}$%? *Ans.* $4,410.

13. If $7\frac{3}{16}$ barrels cost $$24\frac{1}{4}$, what will 1 barrel cost? *Ans.* $3\frac{43}{115}$.

ANSWERS.

Answers are given to all the written and the more difficult of the oral examples. The references are to the page. When two articles with examples of like numbers are on the same page, the references are to both page and article. The number on the left of each column is the number of the example, that on the right is the answer.

P. 39.

1. 177.
2. 189.
3. 209.
4. 270.
5. 93.
6. 144.
7. 220.
8. 202.
9. 191.
10. 128.
11. 209.
12. 242.

P. 40.

13. 1,358.
14. 758 da.
15. 1,109 qrt.
16. 1,688.
17. 1,622.
18. 1,117 yr.
19. 1,470 qrt.
20. 21,953 lbs.
21. 1,633.
22. 2,221.
23. 2,264.
24. 2,638 da.

P. 41.

25. 3,075 ft.
26. 3,373 yds.
27. 1,496 lbs.
28. 2,760 in.
29. $2,621.
30. 371,778.
31. $74,719.
32. 129,479 yds.
33. 298,945 lbs.
34. 189,911 ft.
35. $144,230.
36. 102,749 in.

P. 42.

8. $1,795.
9. $137.
10. 370 mi.

P. 43.

WRITTEN.

1. $1015.
2. $8290.
3. 3,911 bu.

P. 44.

4. 3,555.
5. 10,148.
6. 1,010 mi.
7. $86,312.
8. $2,800.

P. 51.

1. 53.
2. 41.
3. 32.
4. 45.
5. 31.
6. 26.
7. 32 yds.
8. 33 yds.
9. $11.
10. 42 lbs.
11. 72 in.
12. 52 ft.

P. 53.

1. 32.
2. 25.
3. 37.
4. 6.
5. 16.
6. 19.
7. 235.
8. 612.
9. 563.
10. 611.
11. 531.
12. 363 lbs.
13. 212 ft.
14. 445 yds.
15. $552.
16. 491 in.

P. 54.

17. 165.
18. $4,557.
19. 1,027.
20. 6,712 ft.
21. 2,774.
22. 1,832.
23. 86,222.
24. 27,087.
25. 4,014.
26. 173.
27. 631.
28. 7,832.
29. $7,675.
30. 96,084 ft.
31. 973,069.
32. $3,489.
33. 66,481 yds.
34. $462.
35. 10,525 yds.
36. 629,629.
37. 4,909 ft.
38. 136,783.
39. 4045.

P. 55.

7. 282 yds.
8. 286 sheep.
9. $159.
10. $256.
11. $1,875.

P. 56.

7. $12,586.
8. 286 sheep.
9. $18,589.
10. $550.
11. 559 sheep.

P. 57.

6. 431 min.
7. $3,258.
8. $3,353.
9. $506.
10. $984.

P. 58.

7. Second, $5.
8. $418.
9. $5,795.
10. $385.

P. 59.

11. 1714.
12. 30,927.
13. 398.
14. 25,933.
15. 6,760.
16. $6,740.

P. 66.

2. 1,272.
3. 512.
4. 3,232.
5. 1,590.
6. 3,332.
7. 9,872.
8. 37,233.
9. 2,502 ft.
10. 12,901 in.
11. $4,470.
12. 6,237 lbs.
13. 25,160.
14. 302,498 lbs.
15. 488,052.
16. $49,374.

P. 68.

3. 266,616.
4. 164,208.
5. 630,975.
6. 131,577.
7. 256,335.
8. 589,506.
9. 74,088.
10. 578,856.
11. 38,914,176.

12. 85,950,000.
13. 3,320,863,272.
14. 816,415,040.
15. 68,959,488.
16. 6,241,519,790.
17. 105,062,176.
18. 294,360,066.
19. 172,637.
20. 77,896.
21. 504,684.
22. 156,618.
23. 247,932.
24. 318,364.
25. 683,769.
26. 624,064.
27. 170,280.
28. 110,982.
29. 4,324.
30. 43,562.
31. 99,924.
32. 239,184.
33. 133,416.
34. 624,635.

P. 69.

1. 6360.
2. 822,600.
3. 4,076,500.
4. 334,000.
5. 262,200 yds.
6. $58,870.
7. 32,885,600.
8. 162,400.
9. 412,800.
10. 434,000.
11. 171,600.
12. $16,227.
13. 1,112,400.
14. 1,150,050.
15. 261,000.
16. 3,771,000.
17. 1,250,000.
18. 11,200,000.

P. 70.

19. 56,700,000.
20. 365,000.
21. 276,800.
22. $1,425,900.
23. 3,090,000 yds.
24. *1,410,000.*
25. *120,000.*
26. 355,000.
27. 2,347,200.
28. 126,000 ft.

P. 71.

21. 3,744.
22. 10,368.
23. 23,128.
24. 1,848.
25. 52,416.
26. 700,728.
27. 1,680.
28. 51,840.

P. 72, Art. 78.

7. $94.50.
8. $156.14.
9. $70.74.
10. $777.

P. 72, A. 79.

6. $328.
7. $136.
8. 86,400 sec.

P. 73.

9. $806.40.
10. $1,432.

P. 73, A. 80.

8. $266.40.

P. 74.

9. 1,104 mi.
10. $23.78.
11. 21,284.
12. 2,116,400.
13. 394.
14. 4,592,496.
15. 246,099.
16. 4,846,800 yds.

P. 82.

1. 157.
2. 103.
3. 223.
4. 342.

P. 83.

6. $64\frac{4}{5}$.
7. $1,587\frac{1}{3}$.
8. $110\frac{2}{9}$.
9. $535\frac{3}{8}$.
10. 2,704.
11. 3,049.
12. $3,229\frac{1}{5}$.
13. 1,281.
14. $12,648\frac{1}{5}$.
15. 283.
16. $386\frac{1}{9}$.
17. $10,292\frac{2}{11}$.

P. 85.

1. $34\frac{4}{25}$.
2. 312.
3. 38.
4. 54.
5. 64.
6. 192.
7. 312.
8. 405.
9. 371.
10. 108.
11. 223.
12. 215.
13. $58.
14. 594.
15. $564.
16. 259 lbs.
17. 118 lbs.
18. 79 yds.
19. 346.
20. 245.
21. $63\frac{128}{214}$.
22. $56\frac{4}{47}$.
23. $94\frac{1}{32}$.
24. $13\frac{5}{285}$.

P. 86.

1. $21\frac{384}{400}$.
2. $5\frac{4121}{6000}$.
3. $204\frac{781}{900}$.
4. $15\frac{64}{2500}$.
5. $17\frac{548}{16000}$.
6. $24\frac{452}{1500}$.

P. 87.

7. 5.
8. 30.
9. 163.
10. 250.
11. $3\frac{82}{160}$.
12. $22\frac{1470}{3000}$.

P. 88.

6. 52 da., $17\frac{1}{3}$ da., 13 da.
7. $37, $74, $111.
8. 12 mi., 24 mi., 48 mi
9. 144 min., 72 min.
10. $283, $141.50.

P. 89.

6. 12 da.
7. $240, $3,120.
8. 338 w.
9. $70.
10. $780, $242, $478.

P. 90.

11. 40 lbs.
12. 63.
13. 2,684 lbs.

P. 91.

14. 97.
15. $4.
16. $780.
17. 46 lbs.
18. 100 A.
19. $1.40 gain.
20. 30 h.
21. 102 da.
22. 860 A., $42.

P. 97.

1. $2 \times 3 \times 5 \times 7$.
2. $2 \times 3 \times 5 \times 11$.
3. $5 \times 7 \times 29$.
4. $2 \times 2 \times 3 \times 13$.
5. $2 \times 5 \times 31$.
6. $3 \times 3 \times 5 \times 11$.
7. $5 \times 5 \times 17$.
8. $2 \times 2 \times 3 \times 5 \times 11$.
9. $3 \times 5 \times 5 \times 13$.
10. $3 \times 3 \times 5 \times 17$.
11. $2 \times 3 \times 5 \times 19$.
12. $2 \times 2 \times 2 \times 3 \times 3 \times 7$
13. $3 \times 3 \times 3 \times 5 \times 11$.
14. $3 \times 5 \times 5 \times 5 \times 7$.
15. $2 \times 3 \times 3 \times 3 \times 17$.
16. $2 \times 3 \times 3 \times 19$.
17. $2 \times 2 \times 2 \times 103$.
18. $2 \times 2 \times 2 \times 79$.
19. $2 \times 2 \times 137$.
20. $2 \times 5 \times 5 \times 7$.

P. 100.

5. 5.
6. 5.
7. $1\frac{14}{83}$.
8. $1\frac{76}{94}$.
9. $1\frac{5}{17}$.
10. $3\frac{1}{5}$.
11. $12\frac{5}{6}$.
12. $27\frac{30}{31}$.
13. $234\frac{2}{3}$.

P. 101.

1. 6.
2. 14.
3. 42.
4. 5.

5. 15.
6. 9.

P. 100.

1. 4.
2. 3.
3. 11.
4. 47.
5. 189.
6. 267.

P. 103.

1. 60.
2. 120.
3. 72.
4. 180.
5. 180.
6. 840.
7. 432.
8. 840.

P. 110.

1. $\frac{3}{4}$.
2. $\frac{2}{7}$.
3. $\frac{5}{7}$, $\frac{3}{5}$.
4. $\frac{4}{5}$.
5. $\frac{13}{8}$.

P. 111.

2. $\frac{3}{5}$.
3. $\frac{7}{37}$.
4. $\frac{13}{18}$.
5. $\frac{11}{15}$.
6. $\frac{23}{25}$.
7. $\frac{23}{25}$.
8. $\frac{13}{31}$.
9. $\frac{3}{5}$.
10. $\frac{55}{13}$.
11. $\frac{144}{125}$.
12. $\frac{11}{9}$.
13. $\frac{6}{13}$.
14. $\frac{314}{7}$.
15. $\frac{21}{1}$.
16. $\frac{12}{1}$.
17. $\frac{13}{14}$.
18. $\frac{31}{32}$.
19. $\frac{3}{5}$.
20. $\frac{13}{17}$.
21. $\frac{17}{19}$.
22. $\frac{9}{13}$.
23. $\frac{11}{15}$.
24. $\frac{10}{39}$.
25. $\frac{23}{25}$.
26. $\frac{56}{305}$.
27. $\frac{8}{13}$.
28. $\frac{25}{33}$.
29. $\frac{377}{969}$.
30. $\frac{9}{14}$.
31. $\frac{407}{561}$.

P. 112.

3. $4\frac{5}{9}$
4. $25\frac{11}{12}$.
5. $43\frac{7}{11}$.
6. $23\frac{8}{15}$.
7. $14\frac{13}{15}$.
8. $63\frac{1}{7}$.
9. $57\frac{11}{18}$.
10. $8\frac{1}{2}$.
11. $7\frac{13}{14}$.
12. $27\frac{5}{11}$.
13. $106\frac{1}{5}$.
14. $100\frac{413}{420}$.

P. 112.

4. $\frac{86}{3}$, $\frac{18}{2}$.
5. $\frac{72}{8}$, $\frac{21}{8}$.

P. 113, A. 111.

6. $\frac{122}{16}$, $\frac{52}{4}$.
7. $\frac{75}{15}$, $\frac{75}{5}$.
8. $\frac{252}{18}$, $\frac{153}{9}$.
9. $\frac{375}{5}$, $\frac{252}{3}$.
10. $\frac{460}{4}$, $\frac{672}{6}$.
11. $\frac{774}{9}$, $\frac{584}{8}$.

P. 113, A. 112.

4. $\frac{222}{9}$.
5. $\frac{2364}{10}$.
6. $\frac{3015}{14}$.
7. $\frac{393}{8}$.
8. $\frac{697}{11}$.
9. $\frac{585}{14}$.
10. $\frac{1557}{16}$.
11. $\frac{1683}{20}$.
12. $\frac{5037}{44}$.

P. 114.

1. $\frac{6}{12}, \frac{4}{12}, \frac{8}{12}$, & $\frac{42}{210}, \frac{35}{210}, \frac{30}{210}$.
2. $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$, & $\frac{126}{315}, \frac{135}{315}, \frac{140}{315}$.
3. $\frac{8008}{8008}, \frac{5096}{8008}, \frac{4928}{8008}, \frac{5720}{8008}$. $\frac{825}{1320}, \frac{720}{1320}, \frac{924}{1320}, \frac{858}{1320}$.
4. $\frac{8}{12}, \frac{6}{12}, \frac{9}{12}$.
5. $\frac{35035}{45045}, \frac{27027}{45045}, \frac{38610}{45045}, \frac{36755}{45045}, \frac{34650}{45045}$.

P. 115.

5. $\frac{9}{24}$.
6. $\frac{199}{180} = 1\frac{19}{180}$.
7. $1\frac{59}{112}$.
8. $1\frac{43}{44}$.
9. $2\frac{11}{24}$.
10. $2\frac{7}{16}$.
11. $2\frac{7}{8}$.
12. $2\frac{43}{90}$.

P. 117.

WRITTEN WORK.

1. $1\frac{7}{8}$.
2. $3\frac{31}{120}$.
3. $1\frac{9}{50}$.
4. $\frac{57}{60}$.
5. $2\frac{11}{30}$.
6. $1\frac{43}{45}$.
7. $1\frac{5}{27}$.
8. $\frac{911}{1120}$.
9. $1\frac{187}{1008}$.
10. $\frac{487}{560}$.
11. $111\frac{1}{8}$.

12. $14\frac{1}{38}$.
13. $19\frac{17}{36}$.
14. $14\frac{11}{180}$.
15. $14\frac{5}{9}$.
16. $15\frac{11}{14}$.
17. $17\frac{7}{12}$.
18. $18\frac{59}{90}$.
19. $8\frac{2}{15}$.
20. $12\frac{9}{40}$.

P. 118.

4. $\$21\frac{35}{72}$.
5. $\$42\frac{5}{16}$.
6. $19\frac{13}{16}$ lbs.
7. $20\frac{1}{16}$ yds.
8. $135\frac{7}{10}$ yds.
9. $7\frac{3}{20}$ tons.
10. $\$43\frac{3}{4}$.
11. $123\frac{263}{360}$.

P. 120.

13. $\frac{2}{105}$.
14. $2\frac{11}{45}$.
15. $3\frac{14}{178}$.
16. $1\frac{1}{4}$.
17. $6\frac{3}{5}$.
18. $1\frac{22}{35}$.

PROBLEMS.

2. $\$1\frac{1}{8}$.
3. $22\frac{5}{8}$ tons.
4. $4\frac{5}{18}$ yds.
5. $4\frac{9}{112}$.
6. James, $137\frac{3}{10}$ A.
   Joseph, $161\frac{3}{10}$ A.
   Daniel, $177\frac{3}{10}$ A.
7. $23\frac{1}{3}$ yds.

P. 121.

8. $\$1\frac{1}{8}$.

P. 122.

6. $8\frac{8}{9}$ cd., 10 cd.
7. $10, $20.
8. $13\frac{1}{2}$ bu.
9. $\$8\frac{8}{11}$, $9.
10. $20\frac{4}{5}$ cts., 26 cts.
11. $\$58\frac{1}{2}$, $\$82\frac{9}{10}$.
12. $42\frac{3}{7}$ bbls., $66\frac{6}{7}$ bbls.

P. 123.

5. $20\frac{5}{8}$.
6. $30\frac{5}{6}$.
7. $42\frac{4}{9}$.
8. 315.
9. $55\frac{5}{7}$.
10. $50\frac{3}{4}$.
11. $399\frac{3}{7}$.
12. $565\frac{2}{5}$.

P. 124.

14. $636\frac{1}{2}$.
15. 114.
16. 1344.
17. 4758.
9. $\frac{9}{14}$.
10. $\frac{14}{81}$.
11. $2\frac{2}{9}$.
12. 48.

P. 125.

13. $\frac{63}{80}$.
14. $\frac{3}{4}$.
15. $\frac{45}{272}$.
16. $\frac{39}{350}$.
17. $\frac{1}{6}$.
18. $15\frac{3}{7}$.
19. $\frac{17}{18}$.
20. $4\frac{1}{6}$.
21. $858\frac{3}{14}$.
22. $20\frac{4}{7}$.
23. $2\frac{1}{3}$.
24. $\frac{3}{28}$.
25. 63.
26. $2\frac{131}{299}$.
29. $\frac{45}{11}$.
30. $18\frac{20}{27}$.
31. 1,090.
32. $3{,}000\frac{2}{3}$.

P. 126.

1. $\$3\frac{3}{4}$.
2. $\$\frac{5}{12}$.
3. $6.56¼.
4. $547.50.
5. $\$10.46\frac{7}{8}$.
6. $\$4.38\frac{3}{4}$.
7. $\$226\frac{7}{8}$.
8. $\$31.71\frac{7}{8}$.
9. $\$78\frac{1}{8}$.
10. $51\frac{3}{8}$.
11. $90.
12. $25.
13. 2 tons.
14. $1.75.
15. $\$5.46\frac{5}{8}$.
16. $\$1\frac{31}{64}$.

P. 128.

1. $1\frac{18}{19}$.
2. $\frac{2}{177}$.
3. $\frac{1}{44}$.
4. $\frac{14}{1953}$.
5. $\frac{561}{19600}$.
6. $\frac{172}{5185}$.
7. $\frac{5}{24}$.
8. $\frac{9}{32}$.

P. 129.

1. $7\frac{8}{11}$.
2. 130.
3. $1333\frac{1}{3}$.
4. 30.
5. $1102\frac{1}{2}$.
6. $14\frac{62}{67}$.
7. $7\frac{1}{32}$.
8. $11\frac{16}{25}$.

P. 130.

WRITTEN WORK.

1. $5\frac{31}{40}$.
2. $\frac{27}{70}$.
3. $\frac{9}{16}$.
4. $14\frac{7}{12}$.
5. $\frac{9}{14}$.
6. $9\frac{13}{18}$.
7. $5\frac{23}{24}$.
8. $\frac{45}{64}$
9. $1\frac{87}{200}$.
10. $149\frac{1}{3}$.

P. 131.

4. $\frac{45}{64}$.
5. $\frac{7}{8}$.
6. $\frac{28}{17}$.
7. $\frac{27}{56}$.
8. $\frac{1841}{1050}$.
9. $\frac{2577}{811}$.
10. $\frac{2}{45}$.
11. $\frac{455}{8456}$.

P. 132.

1. $7\frac{5}{11}$ bbls.
2. $31\frac{5}{18}$ bags.
3. $\$6\frac{11}{48}$.
4. $13\frac{5}{8}$ yds.
5. $\$3.61\frac{1}{32}$.
6. $197\frac{3}{4}$ lbs.

P. 133.

7. $\$56\frac{1}{4}$.
8. $76\frac{1}{8}$ da.
9. $\$20\frac{1}{8}$.
10. $\$24\frac{5}{16}$.

P. 139.

1. .5601.
2. 1.0193.
3. .6127.
4. 13.326.

P. 140.

5. 37.2863.
6. 15.2646.
7. 466.0942.
8. 66.8155.
9. 2.9775.
10. 22.359.
11. 313.7279.
12. 161.763.
13. 449.881.
14. 85.855.
15. 57.32.
16. 11.9629.

PROBLEMS.

3. $375,445.
4. $10,093,144.

P. 142.

9. 16,439.
10. 5,318.
11. 9.0855.
12. 1.7866.
13. 0.444.
14. 2.108.
15. 3.2756.
16. 94.25.
17. 232.694.
18. 1,313.7638.
19. 7.9437.
20. 63.63.

PROBLEMS.

3. 2,598.96.
4. 54.4188.
5. 574.755.

P. 144.

1. 15.04.
2. 2.544.
3. 40.02.
4 1.0812.
5. 2.6325.
6. 39.442.
7. .0036.
8. 7.488.
9. 215.7.
10. 25.3825.
11. 1.090824.
12. 7.3926.
13. .27846.
14. .01716.
15. 172.86.
16. 24.75.
17. 13.52.
18. 568.944.
19. 125.6334.
20. 698.6315.
21. 3152.9696.
22. 7.8.
23. 842.
24. 473.
25. 650.

P. 145.

PROBLEMS.

1. $15.05.
2. $7.258.
3. $134.075.
4. 22.932 mi.
38.5875 mi
5. 1491.655.
1638.175.
6. $211.736.
7. $16.936.
8. 391.82.
462.275.
9. $160.312.
$218.025.
10. $1.1509.
$2.512.
11. 132.957 mi.
12. $1.912.
$4.556.

P. 147.

1. 2.96.
2. 14.96.
3. .016.
4. 1.6125.
5. 800.
6. 12.4.
7. 632.8.
8. .075.
9. .25.
10. .33.
11. 2000.
12. 56950.
13. 3.741.
14. 3.416.
15. .8437.
16. .8386.

P. 148.

1. $\frac{32}{1000}$.
2. $3\frac{112}{1000}$.
3. $7\frac{2}{1000}$.
4. 70, $\frac{3}{100}$.
5. $\frac{12}{100000}$.

P. 149.

1. .875.
2. .6875.
3. .6.
4. .44.
5. .714+.
6. .782+.
7. .85.
8. .818+.
9. .636+.
10. .692+.
11. .235+.
12. 6.1875.
13. 18.153+.
14. 10.363+.
15. .214+.
16. .882+.

PROBLEMS.

1. $1.25, $2.50.
2. $5.50, $13.75.
3. 9.4 mi., 18 8
mi.

P. 150.

4. $0.375, $0.75,
$1.875.
5. 20.79 A.,
21.415 A.
6. 92.67 mi.
7. $1.45.
8. 161.24 mi.
9. 11.5 cts.,
42.55 cts.
10. 5 hrs.

P. 154.

5. $120.992.
6. $194.725.
7. $77.073.
8. $47.838.
9. $12.73, $41.95.
10. $5.889.
11. $17.46.
12. $34.53.
13. $25.19.
14. $8.685.
15. $4.49.
16. $81.35.
17. $42.75.
18. $164.125.
19. $2,396,925.
20. $1,029.07.
21. $13.14.
22. $87.145.
23. $26.
24. $33.33.

P. 156.

1. 100 s., 160 s.
2. 216d., 132d.
3. £2, £3.
4. $48,665,
$38,932.
5. 5 sov., 6 sov.
6. 600 ct.
7. 50 dc.
8. 2.5 fr.
9. $27.02.
10. 103,626+ fr.
11. $19.466.
12. $14.5995.
13. $1255,557.
14. $241.25.
15. $15.937+

P. 160.

1. 8 m.
2. 8 m.
3. 1.2 m.
4. 1500 m.

P. 161.

5. 20,000 m.
6. 5,000,000 m.
7. 300,000,000 m.
8. 2 sq. m.
9. .3 sq. m.
10. .006 sq. m.
11. 150 sq. m.
12. 270,000 sq. m.
13. 5 st.
14. 1.5 st.
15. 3 st.
16. 40 st.
17. 200 st.
18. 5 l.
19. 12 l.
20. 2 l.

21. 3 l.
22. 4 l.
23. .2 dec. g.
24. 50,000 g.
25. 40 c. g.
26. 6 kil'g.
27. 500 g.
28. 5,156 kil'g.

P. 162.

2. $27.50.
3. $93.75.
4. $5.15.
5. $12.562+.
6. $39.006+.
7. $91.057+.

P. 163.

3. $0.555.
4. $0.75, $2.25.
5. $0.625, $10.315, $16.25
6. $4.16.
7. 125 lbs.
8. $43.125.

P. 164.

2. 117 yds., 256.44+ yds.
3. 39 bbls., 50.24+ bbls.
4. 17.25 wks.
5. 310.84+ yds., 775.19+ yds.
6. 29 bbls., 100 bbls.
7. 137.5 lbs., 275 lbs.
8. 27 bu., 81 bu.

P. 165.

2. $10.137+.
3. $30.59.
4. $16.20.
5. $28.125.
6. $6.48.

P. 166.

2. $28.476.
3. $64.414+.
4. $65.408.
5. $59.416.
6. $2,326+.
7. $88.04.
8. $44.155.

P. 167.

2. $13.33⅓.
3. $21.75.
4. $29.40, $36.
5. $21, $56.
6. $24.
7. $47, $165.75.
8. $14.75, $7.37½.

P. 168.

2. 30.
3. 176.
4. 108 lbs., 200 lbs.
5. 15 qts.
6. 135 bu.

P. 170.

1. $26.60.
2. $257.05.
3. $294 Dr.

P. 181.

10. 18 sc., 36 sc.
11. 12 pks.
12. 70 min.
13. 14 pts., 20 pts.
14. 180 d., 510 d.
15. 180°, 360°.

P. 182.

1. 805 dwt.
2. 9,484 grs.
3. 475 grs.
4. 2,260 grs.
5. 1,444 oz.
6. 13,079 lbs.
7. 1,228 hrs.
8. 6,762 min.
9. 1,670,400 sec.
10. 3,345 in.
11. 9,246 ft.
12. 6,130$\frac{2}{25}$ in.
13. 32,300 li.
14. 10,962 sq. ft.
15. 35,840 A.
16. 200,074 sq. li.
17. 34,728 cu. in.
18. 316 pts.
19. 361 pts.

P. 183.

16. 16 hrs.
17. 8 min.
18. 40 doz.

P. 184.

2. 2 lbs. 16 dwts. 7 grs.
3. 3 lb. 7 oz. 16 dwts.
4. 3 ℥ 1 ʒ 11 grs.
5. 3 qrs. 24 lbs. 10 oz.
6. 4 T. 8 cwt. 1 qr. 17 lbs.
2 T. 4 cwt. 0 qr. 10½ lbs.
7. 5 da. 7 hrs.
8. 4 wks. 15 hrs.
2 wks. 12 hrs.
9. 21 yds. 2 ft. 6 in.
42 yds. 5 ft.
10. 2 mi 257 rds.
5 mi. 194 rds.
11. 79 yds. 2 ft. 7 in.
15 yds. 2 ft. 11 in.
12. 7 bu. 3 pk. 7 qt.
3 bu. 3 pk. 7½ qt.
13. 11 bu.
22 bu.
14. 105 gal.
52 gal. 2 qts.
15. 266 gal. 2 qt. 1 pt.
66 gal. 2 qt. 1 pt. 1 gi.
16. 2° 27′ 24″.
1° 13′ 42″.

P. 184.

17. 18 cu. yds.
18. 24 cd.
19. 36 A.

P. 186.

1. £10 10s.
2. 7 ʒ 2 ℈ 6 grs.
3. 18 cwt. 3 qr. 13 lb.
4. 14 wks. 4 hrs.
5. 12 yds. 8 in.
6. 76° 13′ 15″.

P. 187.

7. 12 T. 16 cwt. 8 qrs.
8. 27 oz. 5 dwts. 19 grs.
9. 24 gal. 1 qt. 1 pt.
10. 34 bu. 3 pks. 3 qts.

P. 188.

5. 124 T. 59 gal.

6. 14 yrs. 37 wks. 4 da. 20 hrs. 58 min. 54 sec.
8. 12 yrs. 3 mos. 24 da.
9. 21 yrs. 4 mos. 20 da.

P. 191.

6. 157 lbs. 2 oz. 1 dwt.
7. 135 cwt. 24 lbs.
8. 211 da. 2 hrs. 51 min.
9. 113° 18′ 31″.

P. 193.

1. 4 da. 3 hrs. 15 m.
2. 4 wks. 5 da. 17 hrs.
3. 3 bu. 2 pks. 3 qts.
4. 5 cwt. 2 qrs. 16 lbs.
5. 6 lbs. 3 oz. 8 dwts. $13\frac{1}{3}$ grs.
6. 1 cwt. $10\frac{3}{13}$ lbs.
7. 5.
8. 8.
9. 11.
10. 5.

P. 198.

2. 8.05 lbs., 1.75 lbs., 3.50 lbs.
3. $34.65, $27.28, $66.
4. 8.32 wks., 3.2 wks., 8 wks.
5. 1030 yds., 1410 yds.
6. 28.8 bu., 120 bu.

P. 199, A. 232.

7. $300, $1008, $940.80.
8. 87 ft., 500 ft., 600 ft.
9. 84 gals., 132.3 gals., 197.4 gals.
10. $36, $300.
11. $3.75, $30, $22.50.

P. 199, A. 233.

2. $405.
3. 707.6 yds.
4. 86.4 A.
5. $237.62.
6. 38.4 T.

P. 199, A. 234.

2. 32 rds.
3. $378.
4. 58.88 wks.
5. 616 yds.
6. $976.50.

P. 200, A. 235.

2. 7%.
3. 5%.
4. 20%.
5. 10%.
6. 1%, $1\frac{1}{99}$%.

P. 200, A. 236.

2. 118.08.

P. 201.

3. $401.78\frac{4}{7}$.
4. 3,300.
5. 124 pupils.

PROBLEMS.

1. 4,875.
2. $3,622.50.
3. 70 chickens. 112 ducks.
4. $36\frac{2}{25}$ gal.
5. $13,000.

P. 202.

4. $74.28+.
5. $47.25.

P. 203.

6. $176.62+.
7. $342.75.
8. $17.50.
9. $25.
10. $1886.19.

P. 204.

2. $96.
3. $680.
4. $827.20.
5. $11,650.
6. $28\frac{2}{5}$ cts.
7. $815.50.

P. 206.

3. $420.
4. $292.82.
5. $148.75.
6. $91.
7. $140.40.
8. $255.
9. $588.
10. $294.
11. $178.
12. $617.10.
13. $1587.20.
14. $2100.40.
15. $2140.352+.
16. $1873.581+.

P. 207.

3. $9.
4. $9.33\frac{1}{3}$.

P. 208.

5. $5.
6. $6.40.
7. \$$13.82\frac{1}{2}$.
8. $156.26.
9. $217.09.
10. $10.26.
11. $143.
12. \$$8106.56\frac{2}{3}$.

P. 209.

1. $1.34+.
2. $36.58+.
3. $2.
4. $0.749+.
5. $.75.
6. $1.797+.
7. $0.675.
8. $0.68+.

P. 211.

3. $224.581+.
4. $136.125.
5. $1182.751+.
6. $836.92.
7. $1381.415.
8. $1418.75.

P. 213.

3. $1,102.08.
4. $1,331.52.

P. 214.

2. $515.816.

P. 216.

1. $438.59+.
2. $33.963.

P. 217.

1. $1,975.58.
2. $2,945.75.

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